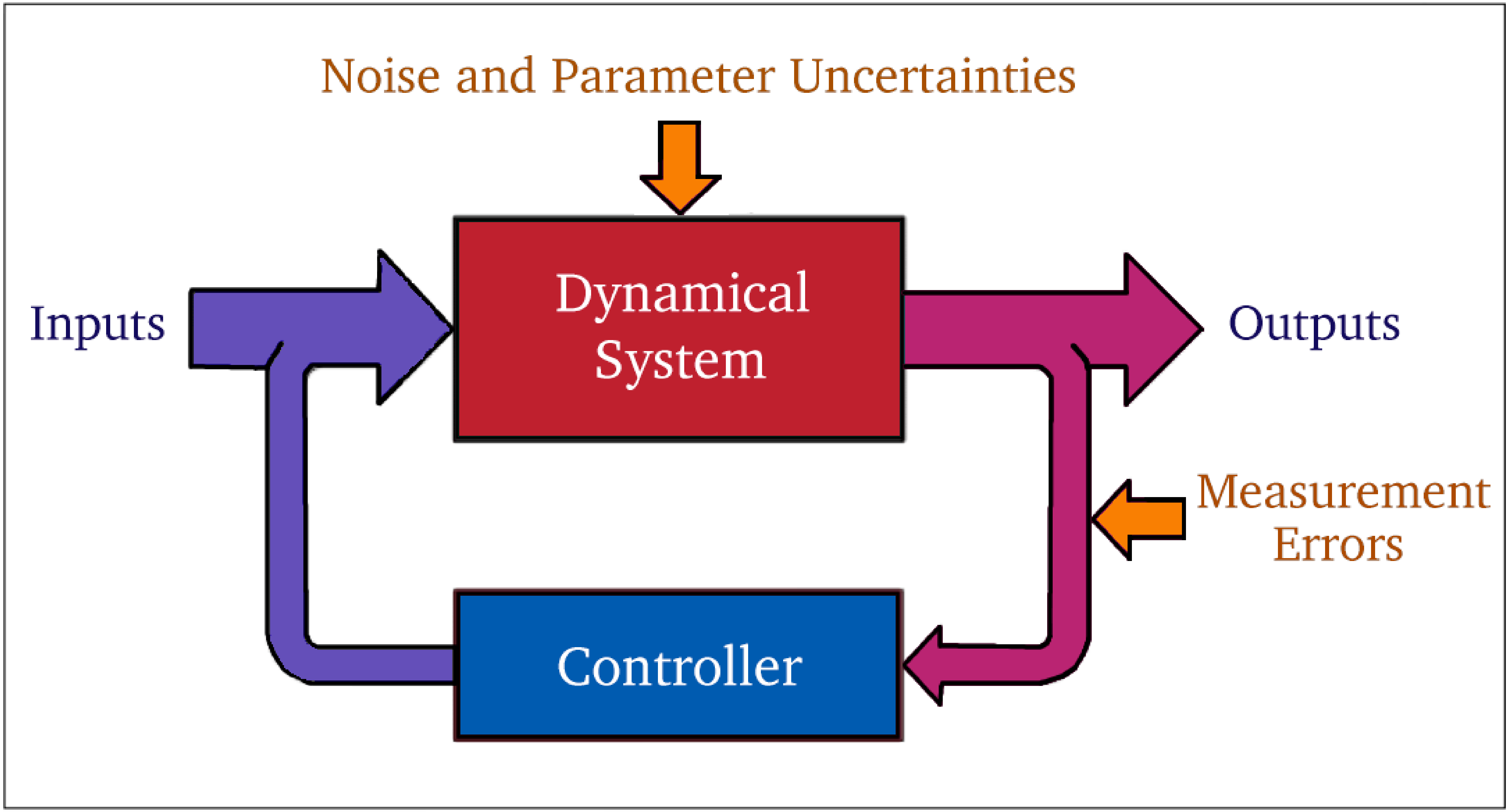
What's Quantum about Quantum Feedback Control?

Kurt Jacobs

Umass Boston, 100 Morrissey Blvd, Boston MA 02125

Controlling Systems with Feedback



Feedback in Quantum Systems...

Measurements **change** the dynamics of a system,
and introduce **noise**.

This means that...

One. Must include measurements when optimizing control

Two. Must choose our measurements carefully: New applications for feedback: ***adaptive measurements*** to reduce noise and optimize information extraction.

Three. The complex interaction between quantum measurements and dynamics also leads to other effects...

Quantum Feedback: Applications

1. Quantum versions of traditional applications...

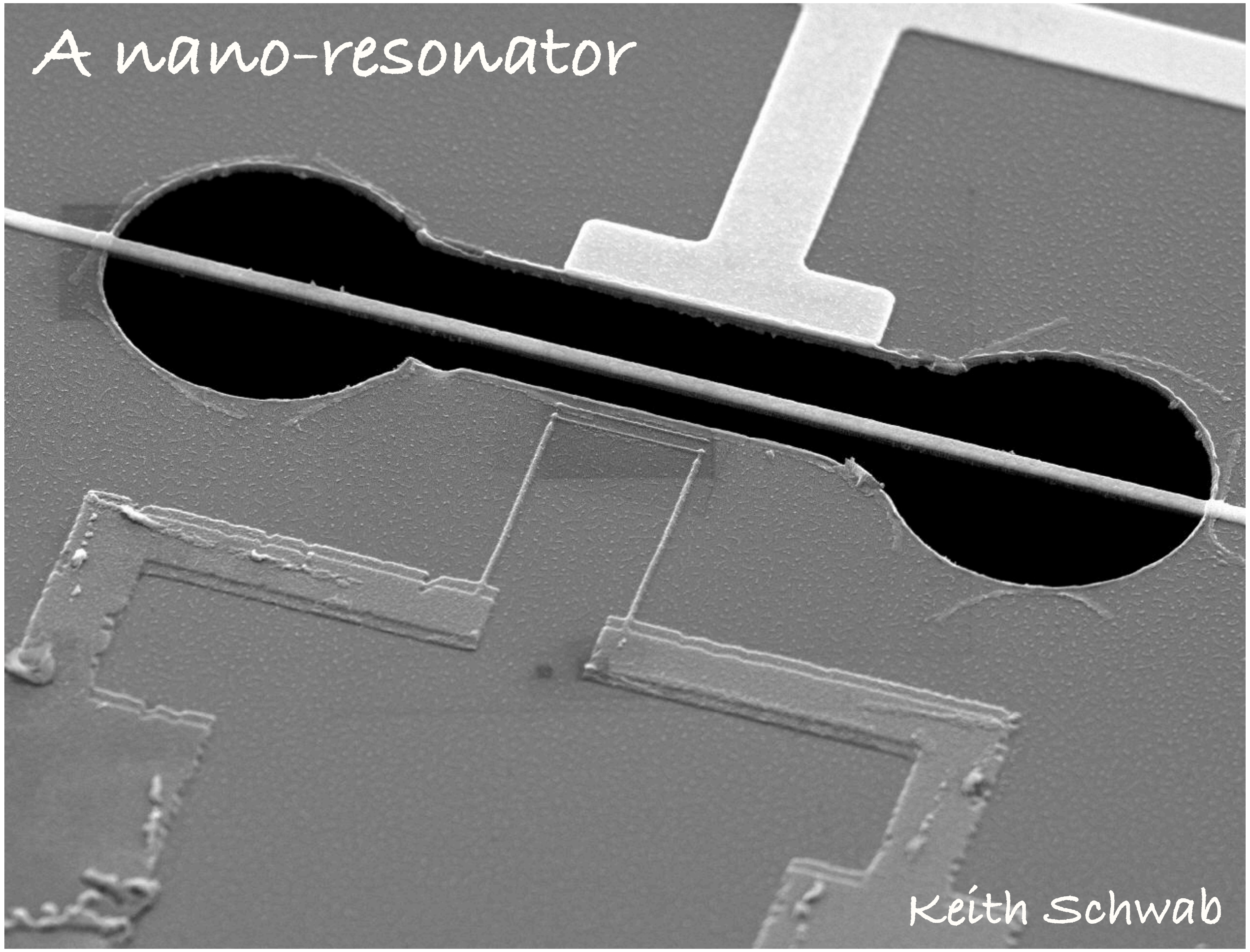
Stabilization/Noise Reduction
Cooling a system to its ground state
Quantum Error-Correction

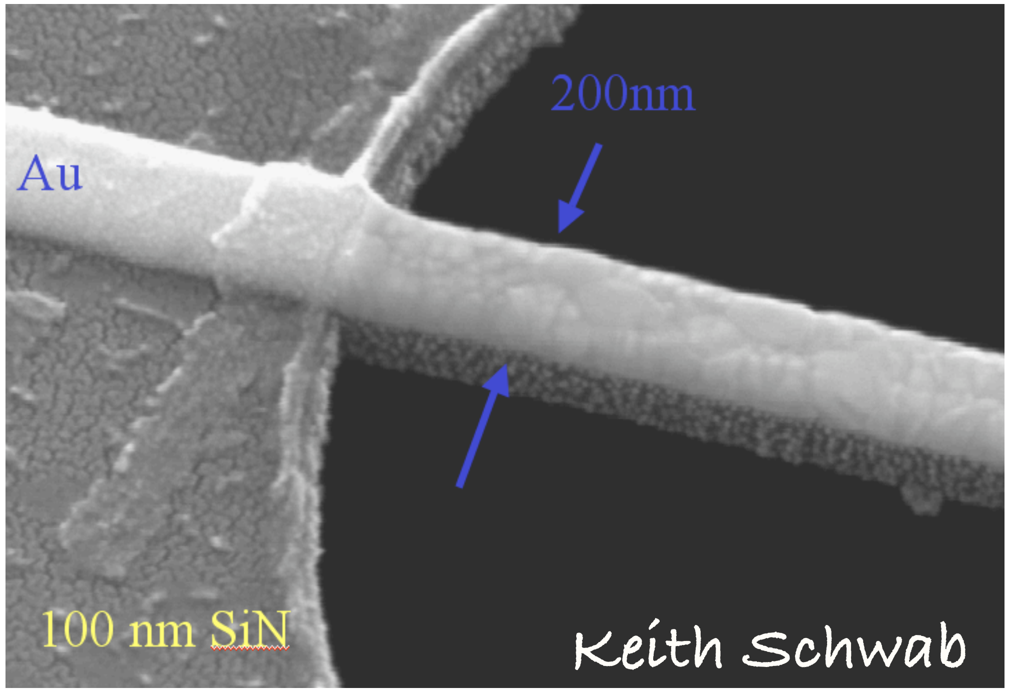
2. Adaptive measurements for...

Communication
Precision measurement
Speeding up State-Collapse/Preparation

3. And even using measurements for control...

Cooling a Nano-Mechanical Resonator





Specs:

Frequency ~ 100 MHz

Quality factor ~ 10^5

— Hopkins, Jacobs*, Habib and Schwab, Phys. Rev. B **68**, 235328 (2003)

* asterisks denote authors who are here at the workshop

Optimal Feedback Cooling

It turns out that...

A Quantum Oscillator + Position measurement + Linear feedback forces	$\equiv$	A Classical Oscillator + Position Measurement + Linear feedback forces + *Extra* white noise

So we can use standard optimal control theory for linear systems to cool and control quantum oscillators.

— Belavkin ~ 1980's, see Rep. Math. Phys. **43**, 405 (1999)

— Yanagisawa & Kimura, in *Learning, Control and Hybrid Systems* (Springer, 1998)

— Doherty & Jacobs*, Phys. Rev. A **60**, 2700 (1999)

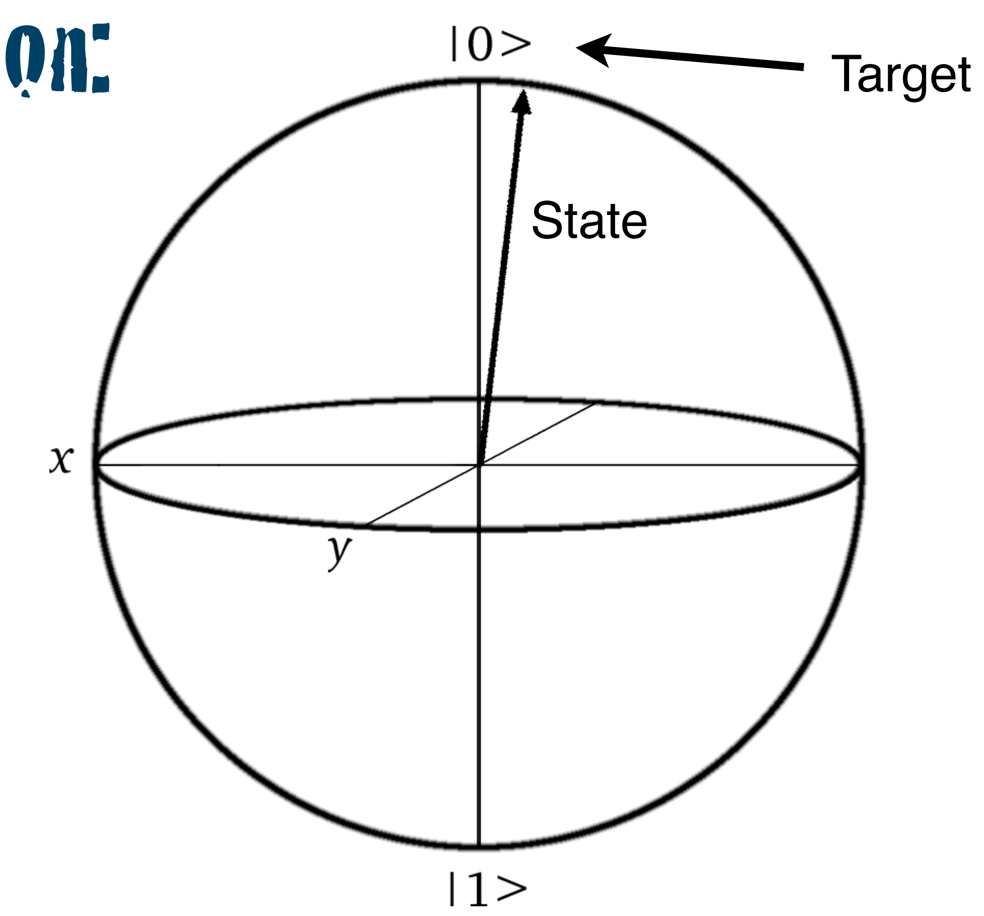
Optimal Linear Control for other systems?

No!

All other quantum systems, when measured, are nonlinear.
Very few optimal results exist for nonlinear systems.

There is one known partial exception:

For a single qubit, in the "regime of good control" (when the control keeps it near the target-state) the motion is nearly linear



—Li* & Jacobs*, Quantum Information & Computation **9**, 395 (2009)

Quantum States of Nano-Resonators

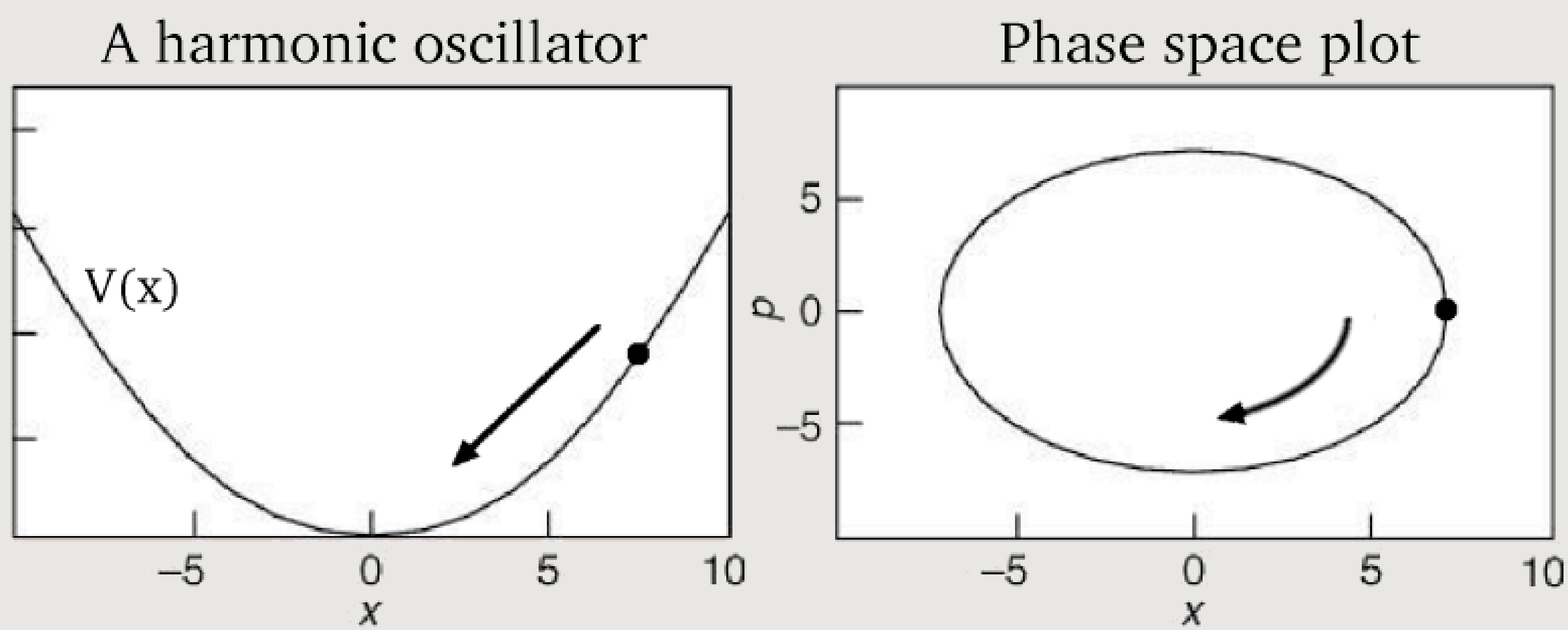
When a resonator is really cold we can create interesting states…

We can create a "quantum superposition" in which the resonator is in "two places at once"

Can do this by continuously measuring the ***square*** *of a resonator's* ***position****.*

— Jacobs*, Tian, and Finn*, Phys. Rev. Lett. **102**, 057208 (2009)

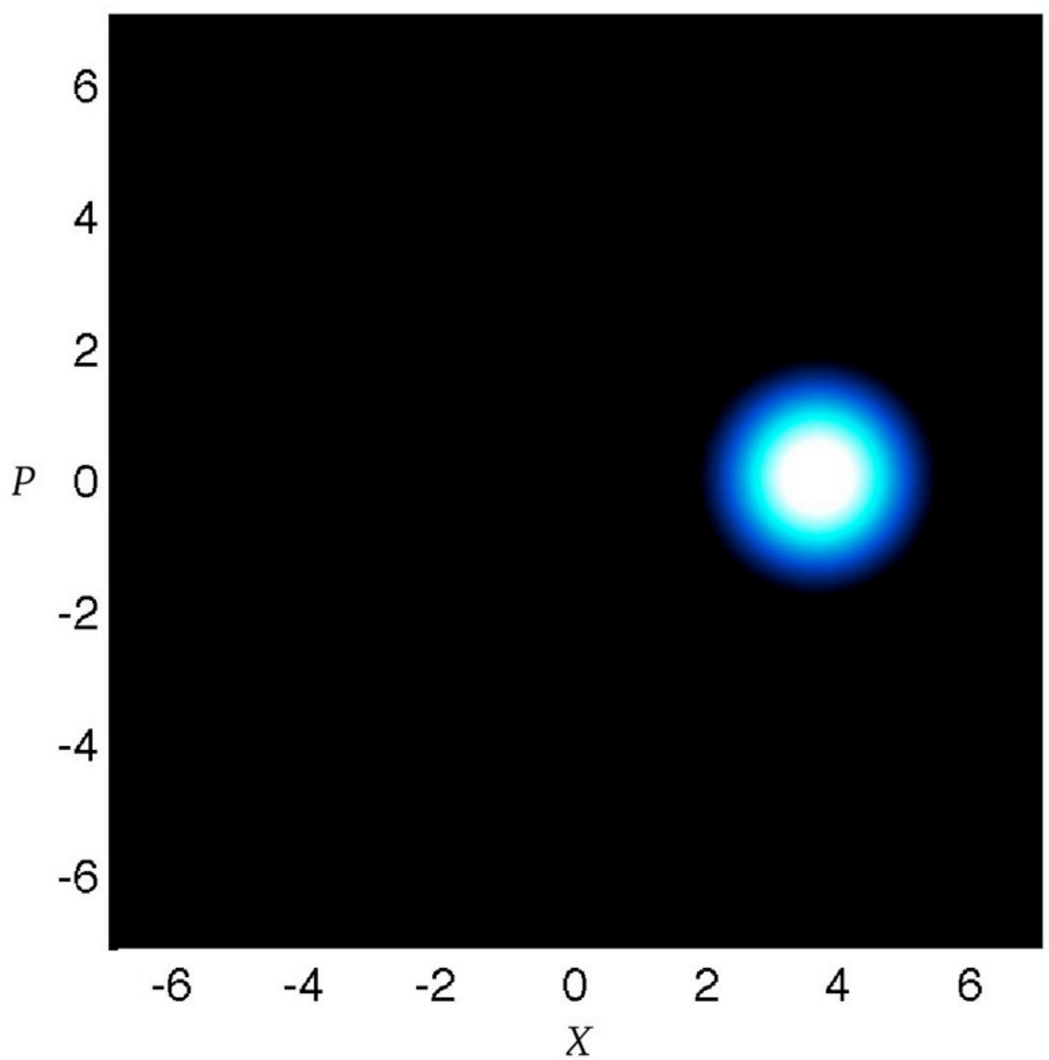
Phase-space plots are useful to visualize this process:



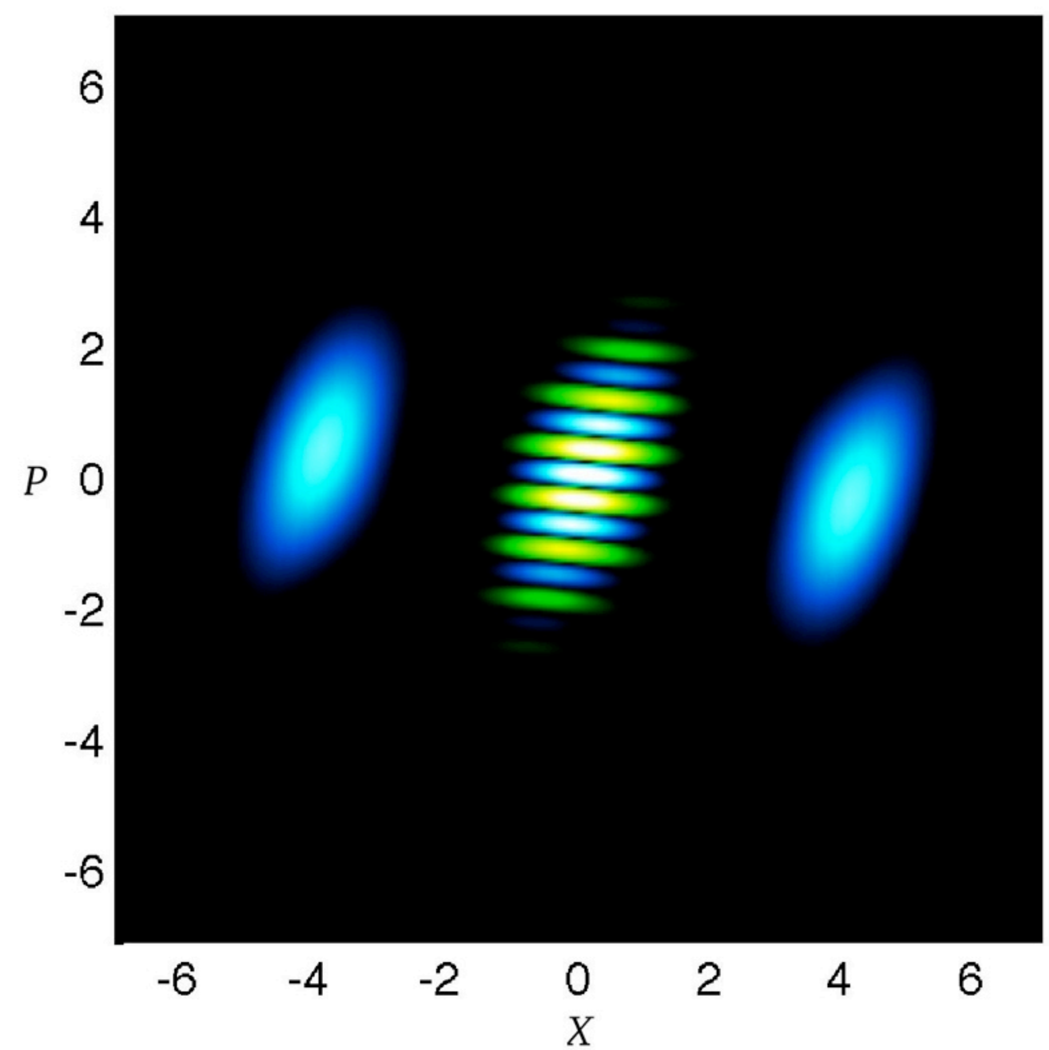
Quantum States of Nano-Resonators

The state of a quantum system can be represented by a "quasi"-probability density over phase-space, **W(x,p)**. This is a **quasi-probability** because it can be negative. The marginals of **W(x,p)** are the real probability densities for **x** and **p**. **W(x,p)** is called the Wigner Function.

A localized 'classical' state

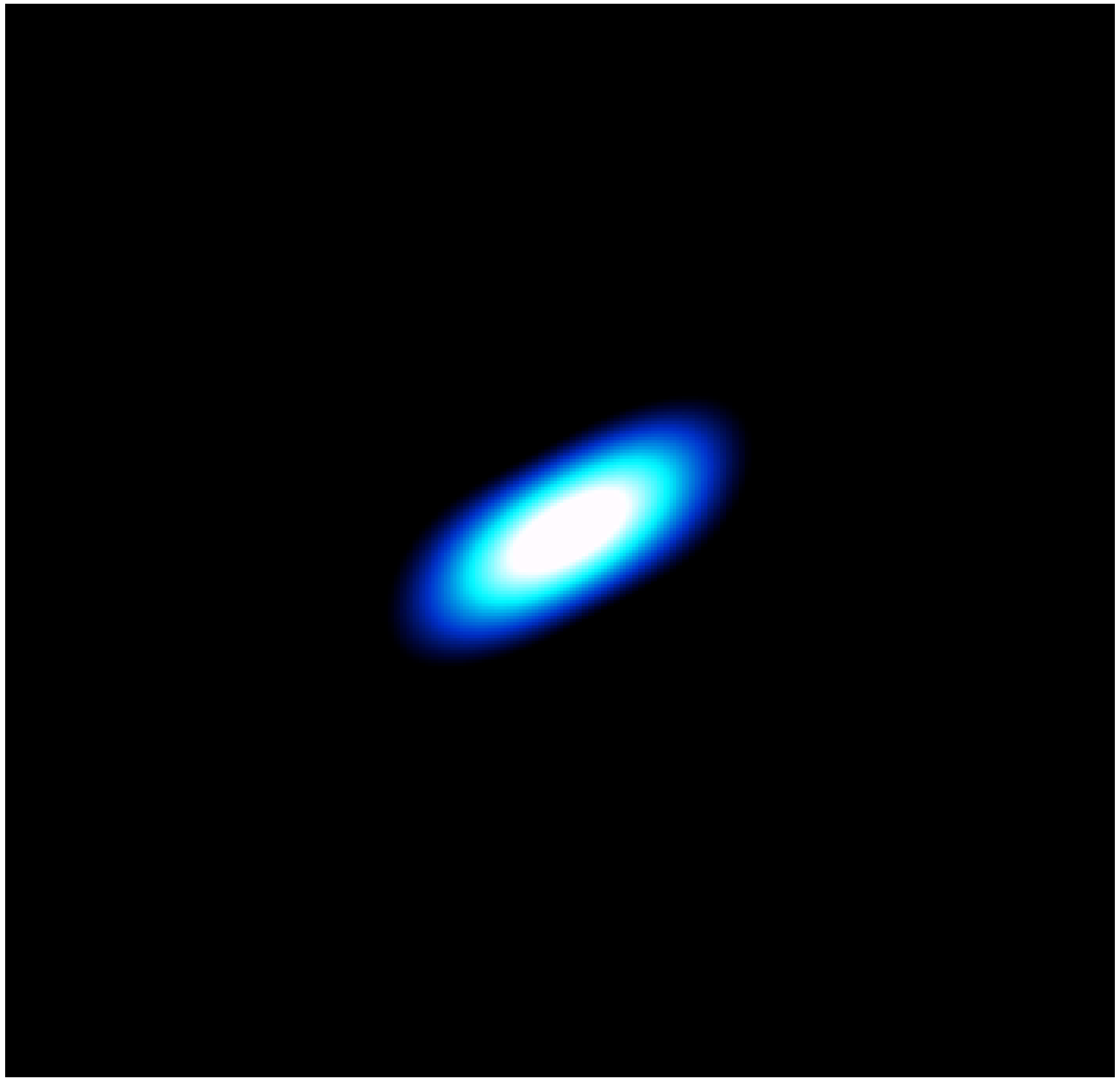


A superposition state



Creating a Superposition (Cat) State

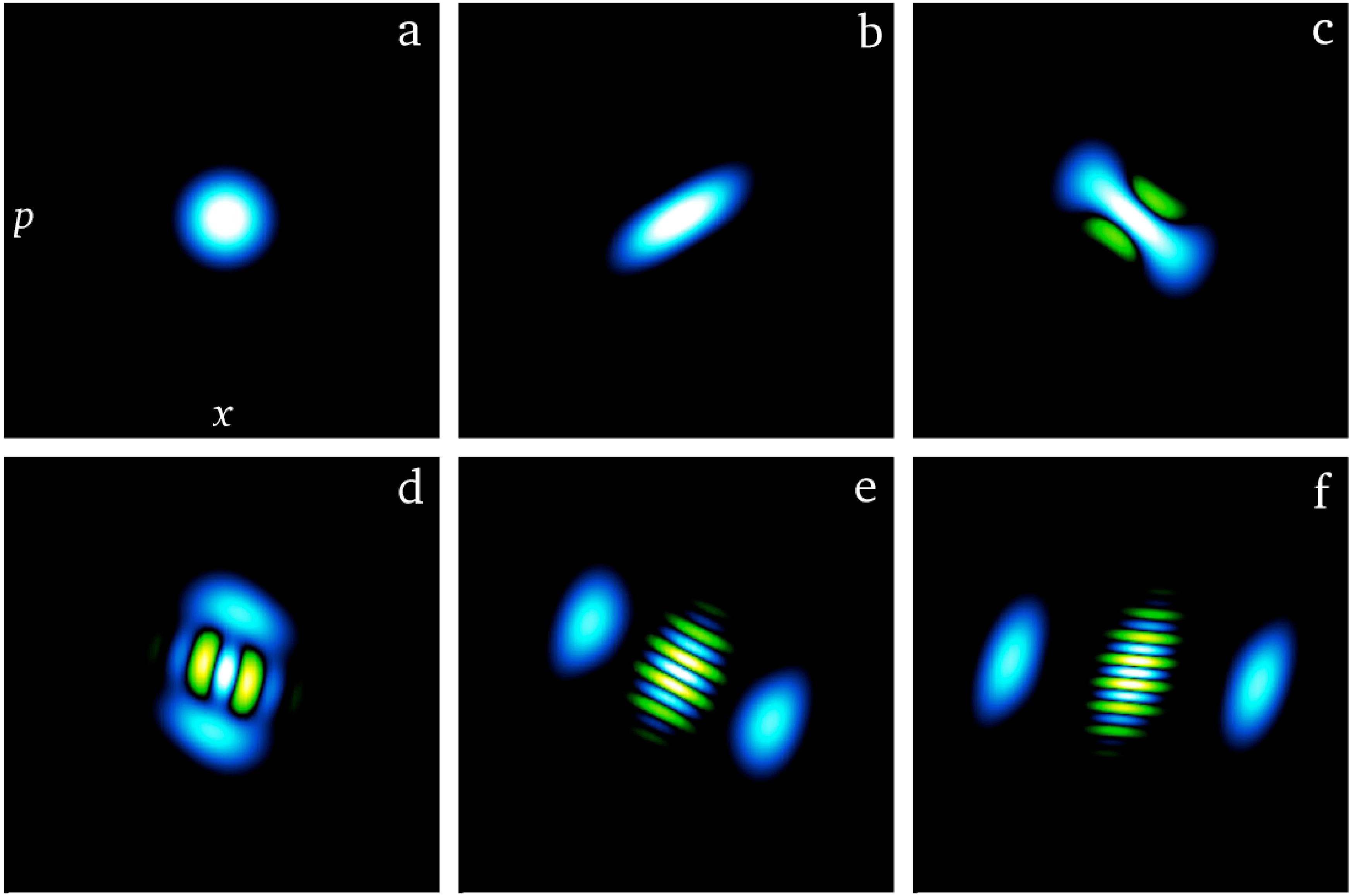
Movie of a continuous measurement creating a 'cat' state:



— Jacobs*, Tian, and Finn*, Phys. Rev. Lett. **102**, 057208 (2009)

Creating a Superposition (Cat) State

Some Snapshots of creating a 'cat' state:

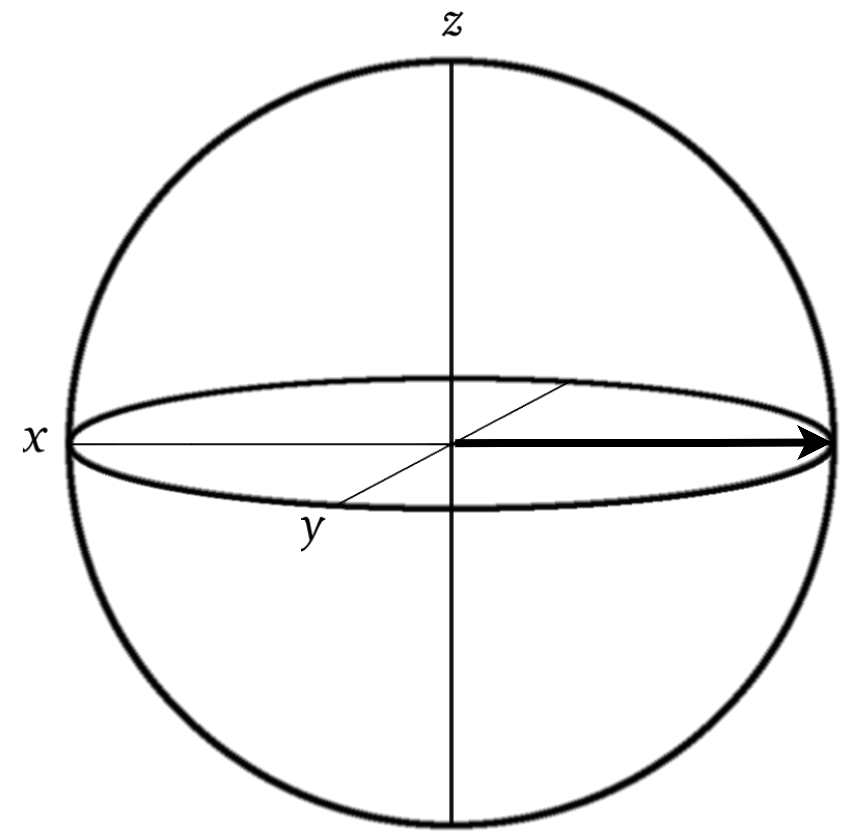


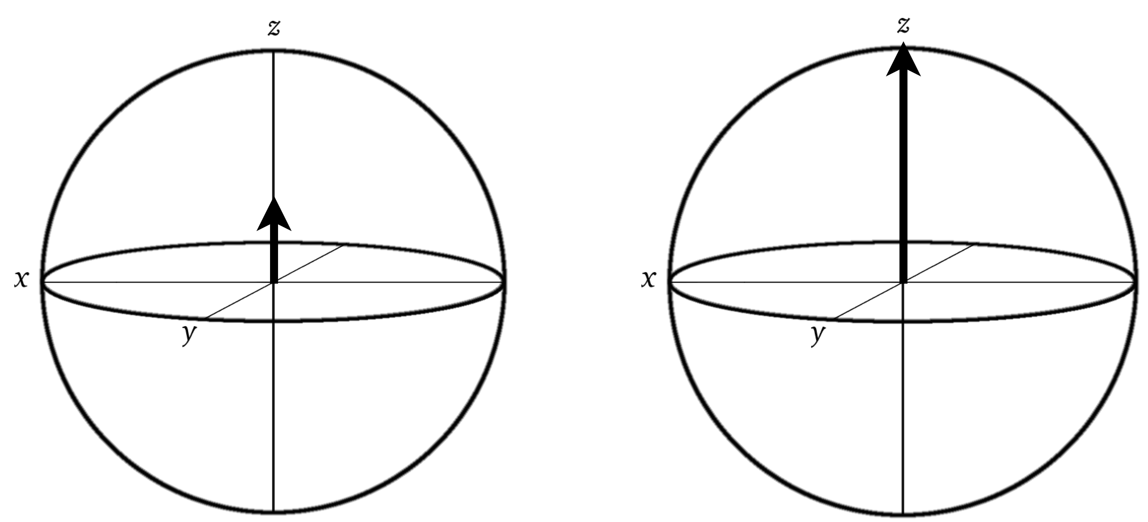
— Jacobs*, Tian, and Finn*, Phys. Rev. Lett. **102**, 057208 (2009)

Spins/Qubits – The Bloch Sphere

A single spin-1/2 can point in any direction, but has only two states in a given direction - when measured along one direction, it will either be found to point in that direction, or directly opposite.

The "Bloch-sphere" is a way of representing the spin state. If the state of the spin is completely known, then the "Bloch-vector" has length 1 and gives the direction of the spin.



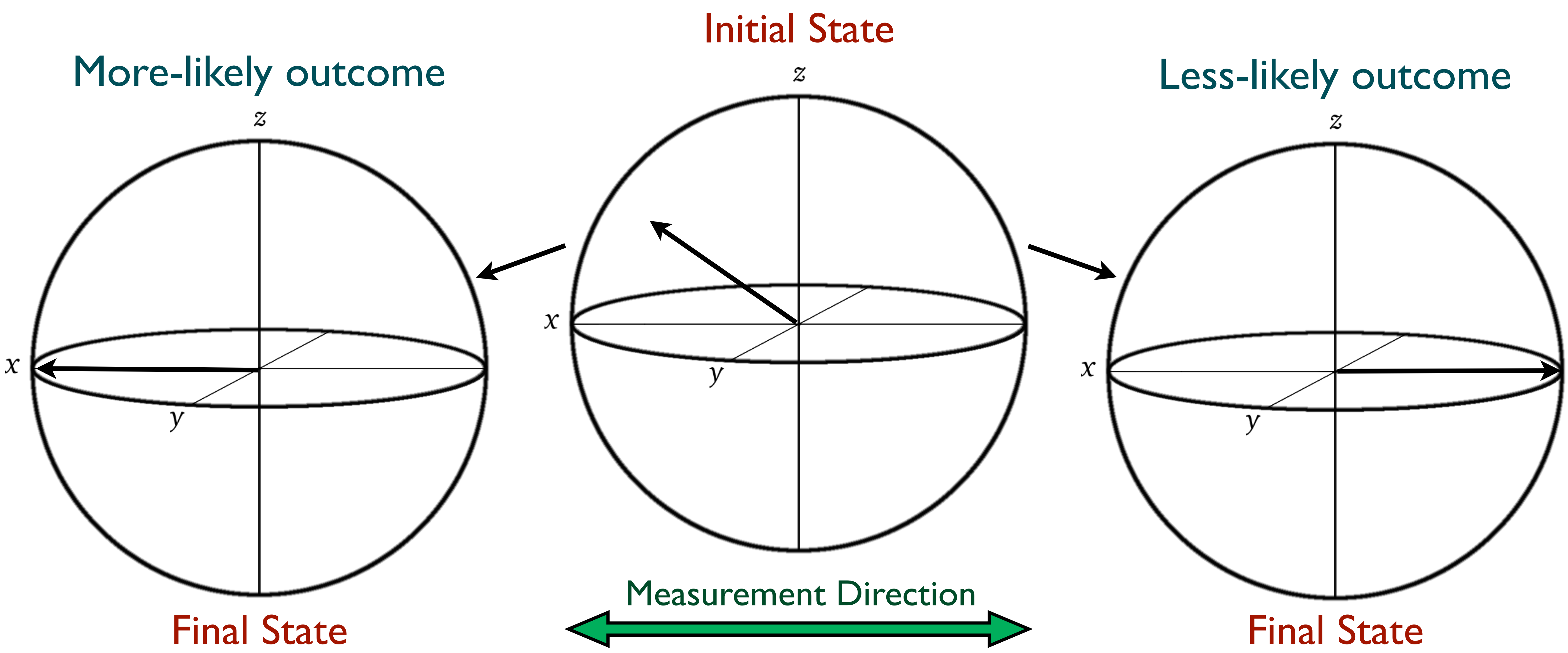


Very uncertain   Certain

If the spin state is not completely known, then the Bloch-vector is shorter. If the spin direction is completely unknown, then the Bloch vector has no length, and so no direction

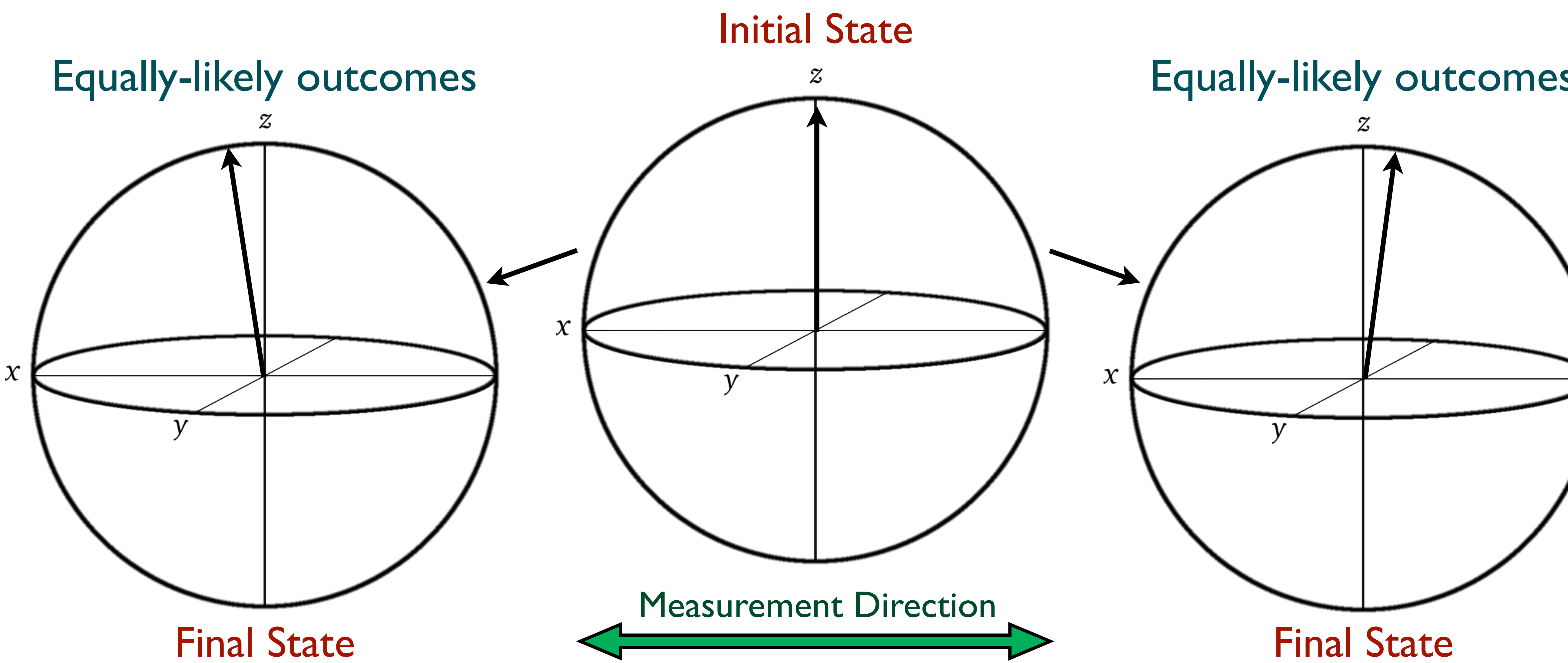
Measurements on Qubits

Quantum measurements change the state of a system. Because of this, quantum measurements provide information about two separate things — the initial state, and the final state. Below is an example in which an initial state of a qubit is strongly measured in the x-direction. The final state is completely determined the measurement, but only partial information is provided about the initial state.



Continuous Measurements on Qubits

In feedback control one usually considers continuous measurements. For short time-intervals these only extract a little bit of information about the state of the system. If we start in a +z state, then the effect of a continuous x-measurement for a short period of time is:



Two Purely Quantum Effects

The dynamics introduced by quantum measurements (the quantum back-action) leads to some remarkable effects. We will describe two here:

1. Feedback can be used to increase the speed at which the measurement "collapses" the system to a final state.

— Wiseman and Ralph New. J. Phys **8**, 90 (2006)

— Combes and Jacobs*, Phys. Rev. Lett. **102**, 057208 (2006)

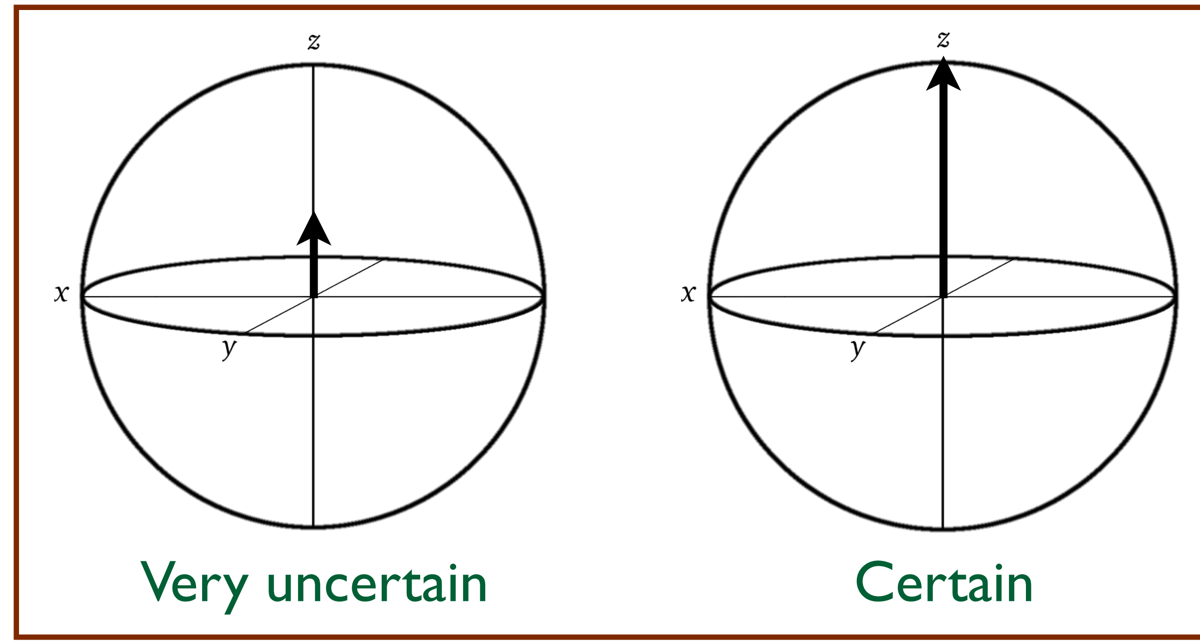
2. The random quantum back-action of adaptive measurements can be used to control a quantum system

— Jacobs, Eprint arXiv::0904.3745 (2009)

(This is hot off the press - was posted on monday!)

Speeding-Up Quantum "Collapse"

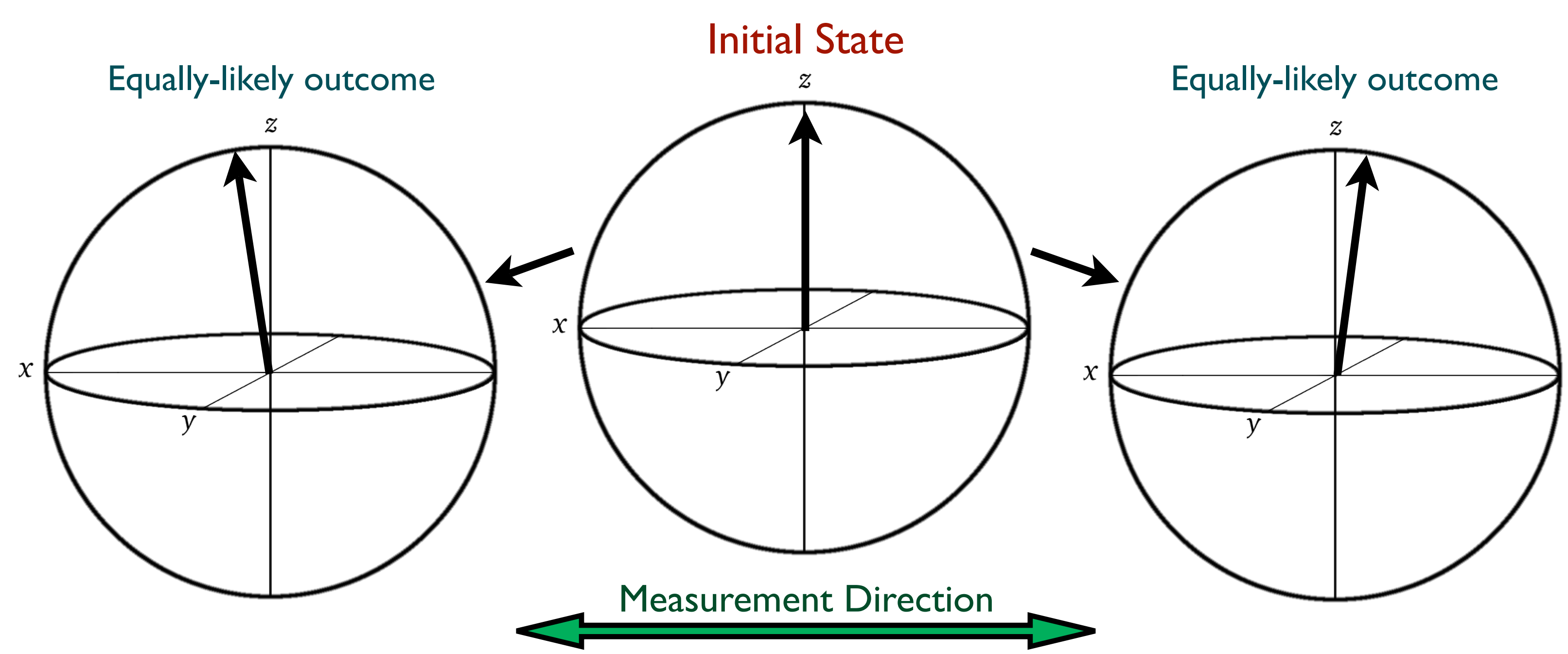
The closer the Bloch-vector to the surface of the sphere, the more certain we are of the spin direction:



1. It turns out that measuring in a direction orthogonal to the spin direction lengthens the Bloch vector faster. This is a purely quantum effect.

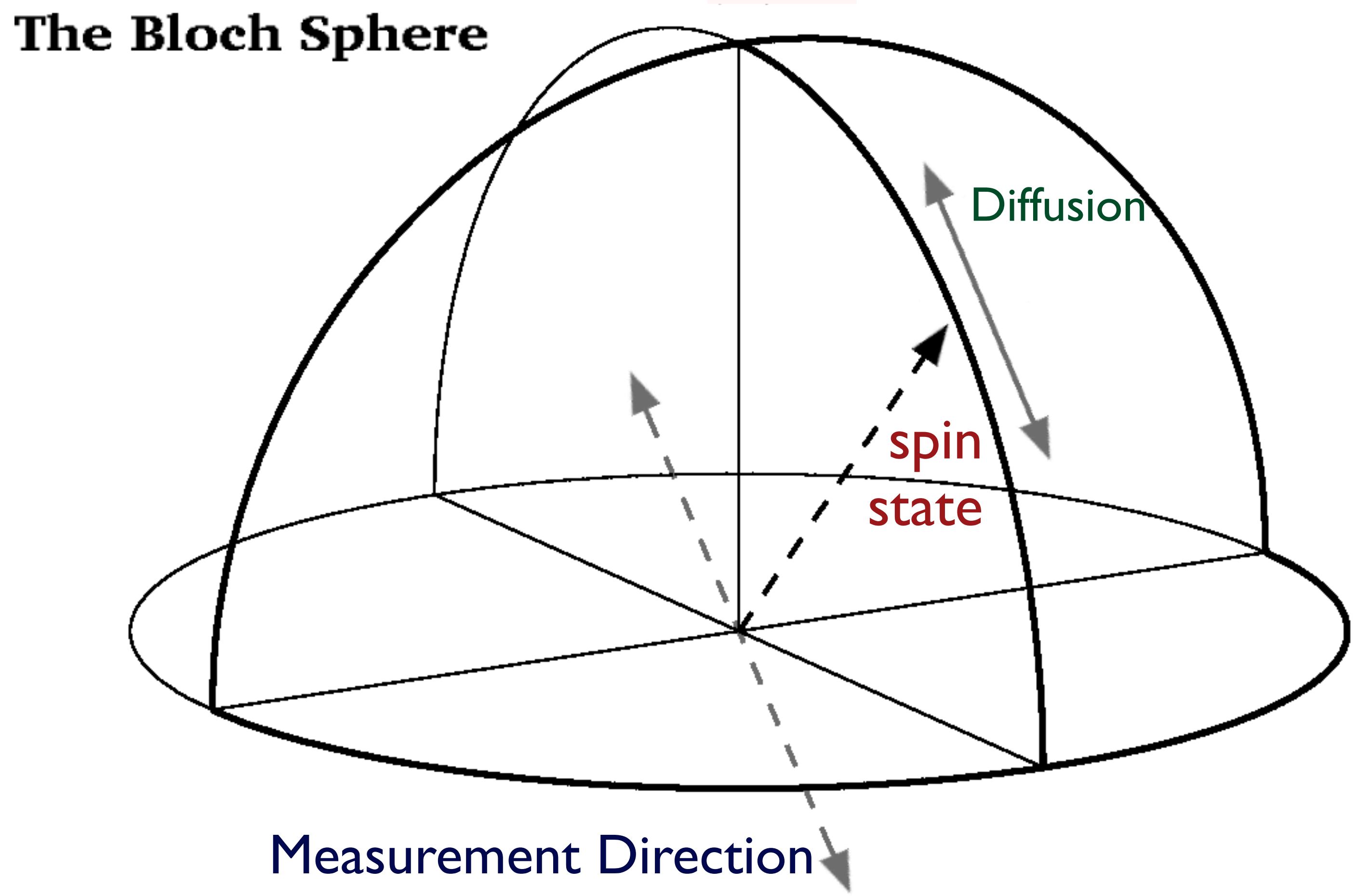
2. But this changes the direction of the Bloch vector

3. So we have to continually change the direction of the measurement to make the state "collapse" fastest to a single certain direction. The diagram illustrates this:



Controlling a system using Back-Action

This is remarkably simple. The random back-action of the measurement causes diffusion on the surface of the Bloch sphere. By using feedback to control the rate and the direction of the measurement, we can set up a diffusion gradient. Diffusion gradients act similarly to forces - a system will move in the direction in which the diffusion decreases. If the measurement is strong, this can be used to obtain very accurate control.



Further Reading

Some introductory articles are:

Jacobs, Proceedings of ASCC06 p.35 (2006)
downloadable at: http://arxiv.org/abs/quant-ph/0605015

Habib, Jacobs, and Mabuchi, Los Alamos Science **27**, 126 (2002)
downloadable at: http://www.quantum.umb.edu/Jacobs/QFC.html

Jacobs & Shabani, Contemporary Physics **27**, 126 (2008)
downloadable at: http://arxiv.org/abs/quant-ph/0605015