Feedback Control in Quantum Systems

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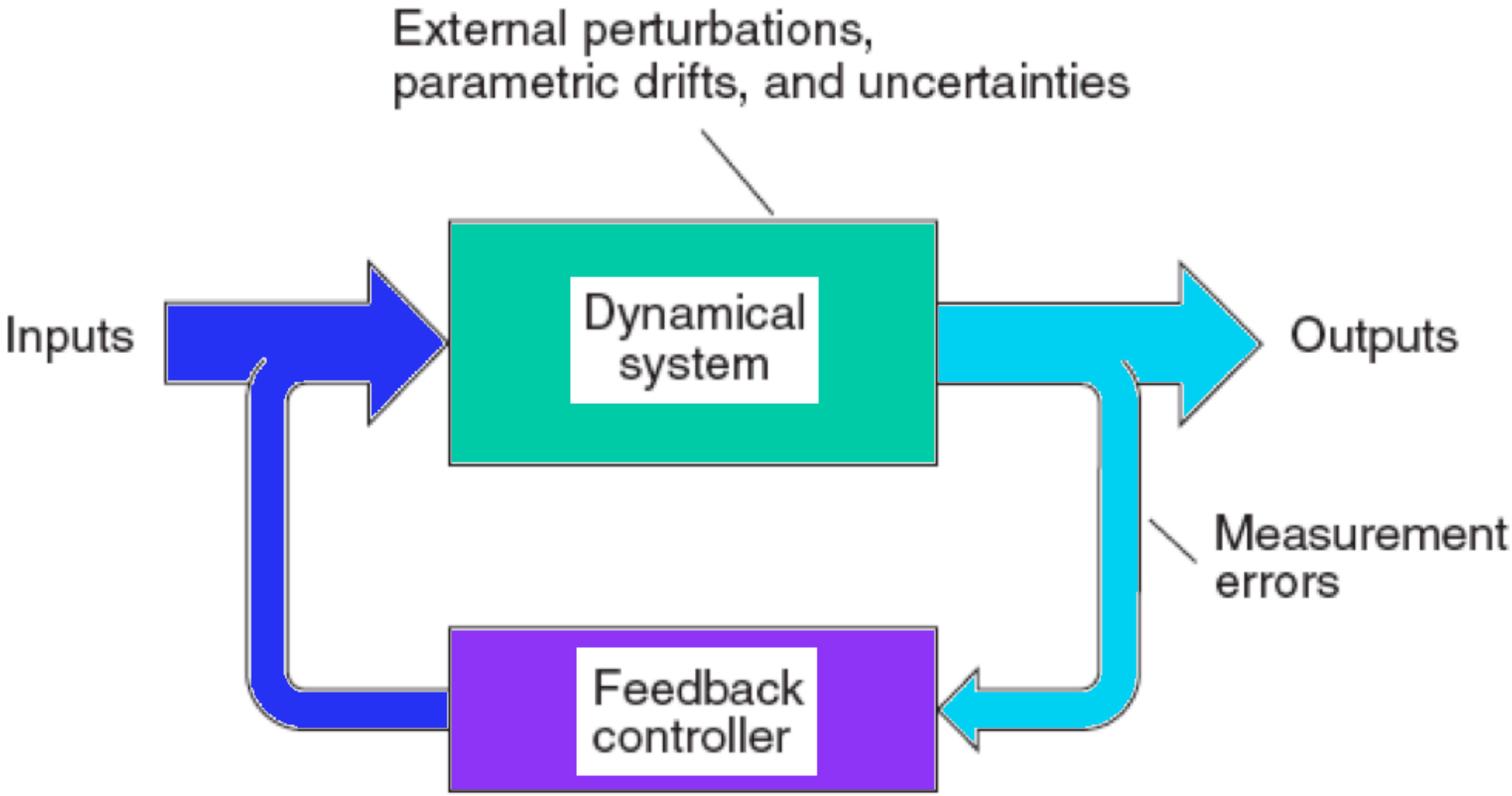
Feedback Control



What's Different about Quantum Control?

Quantum Systems are Dynamical Systems

---- In this sense control theory applies directly to quantum systems

However, two key points distinguish quantum feedback control

1. Measurement *changes* the dynamics of a system

2. Measurement induces noisy and *non-linear* dynamics

These differences also lead to new applications

Applications of Quantum Feedback Control

1. Traditional applications:

   --- Noise reduction and stabilization: e.g. Cooling

2. Adaptive measurement:

   --- Maximizing information and minimizing disturbance

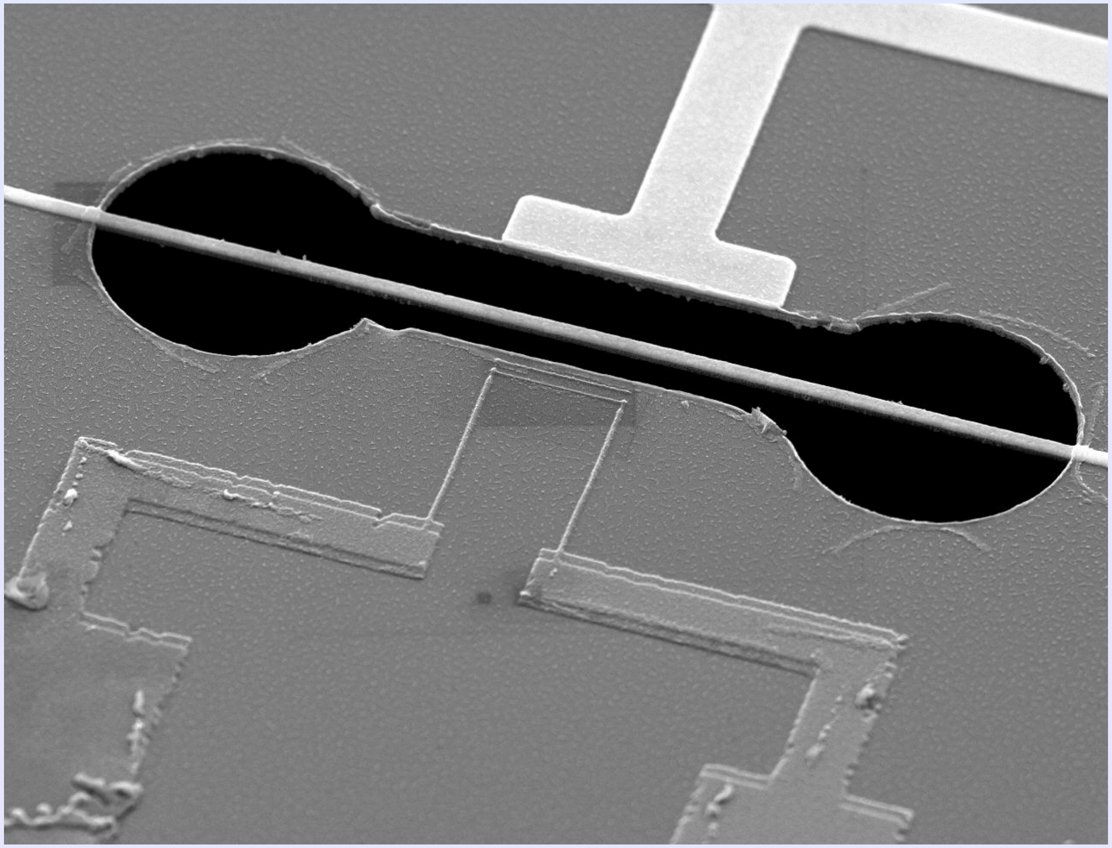
3. Quantum error correction:

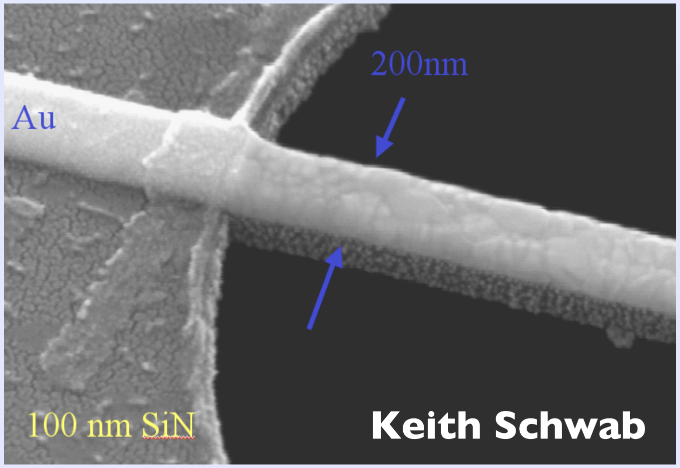
   --- Correcting errors while hiding information

Cooling a Nano-Mechanical Resonator

--- Hopkins, Jacobs, Habib and Schwab, Phys. Rev. B 68, 235328 (2003)

A Nano-Mechanical Resonator:





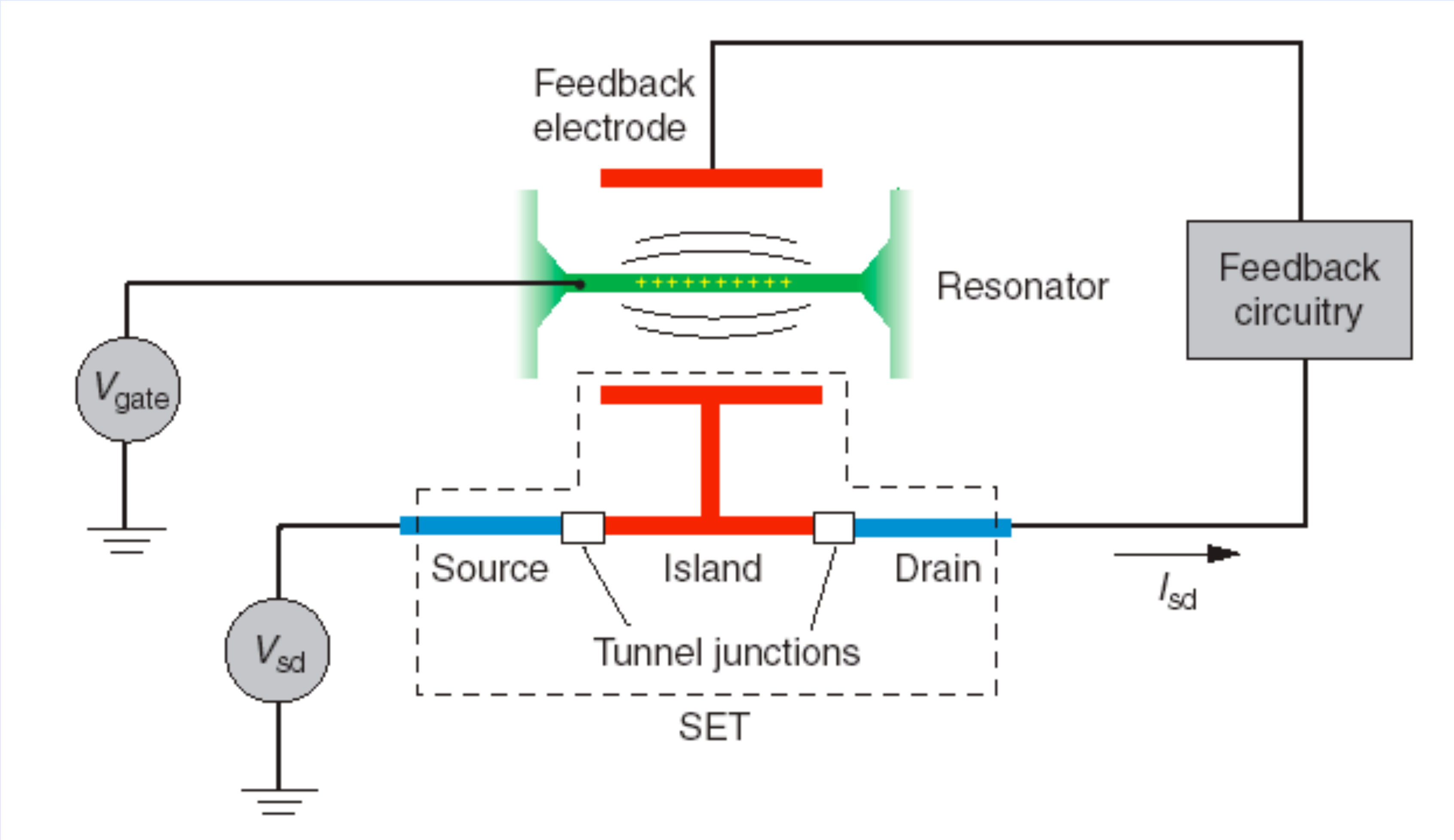
Parameters:

$\omega \sim 10 - 1000\ Mhz$

$Q \sim 10^5$ for lower range of frequencies

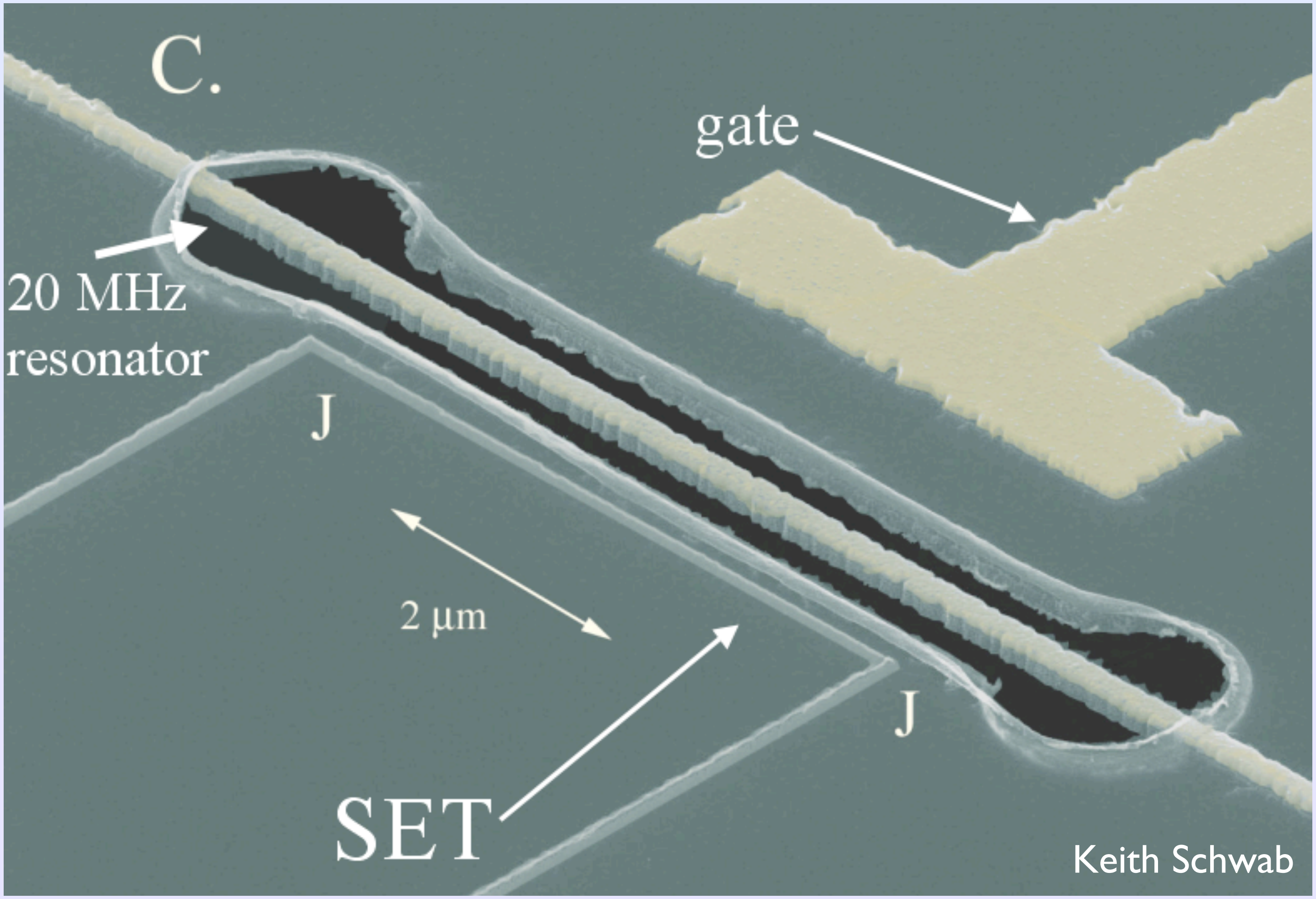
To see quantum effects, need $k_B T < \hbar\omega \quad \rightarrow \quad T \approx 1\ \text{mK}$

Implementing Measurement and Feedback



- Use a single electron transistor (SET) to monitor the position of the oscillator
- Use a voltage gate to apply a feedback force to the oscillator

Implementing Measurement and Feedback



- Use a single electron transistor (SET) to monitor the position of the oscillator
- Use a voltage gate to apply a feedback force to the oscillator

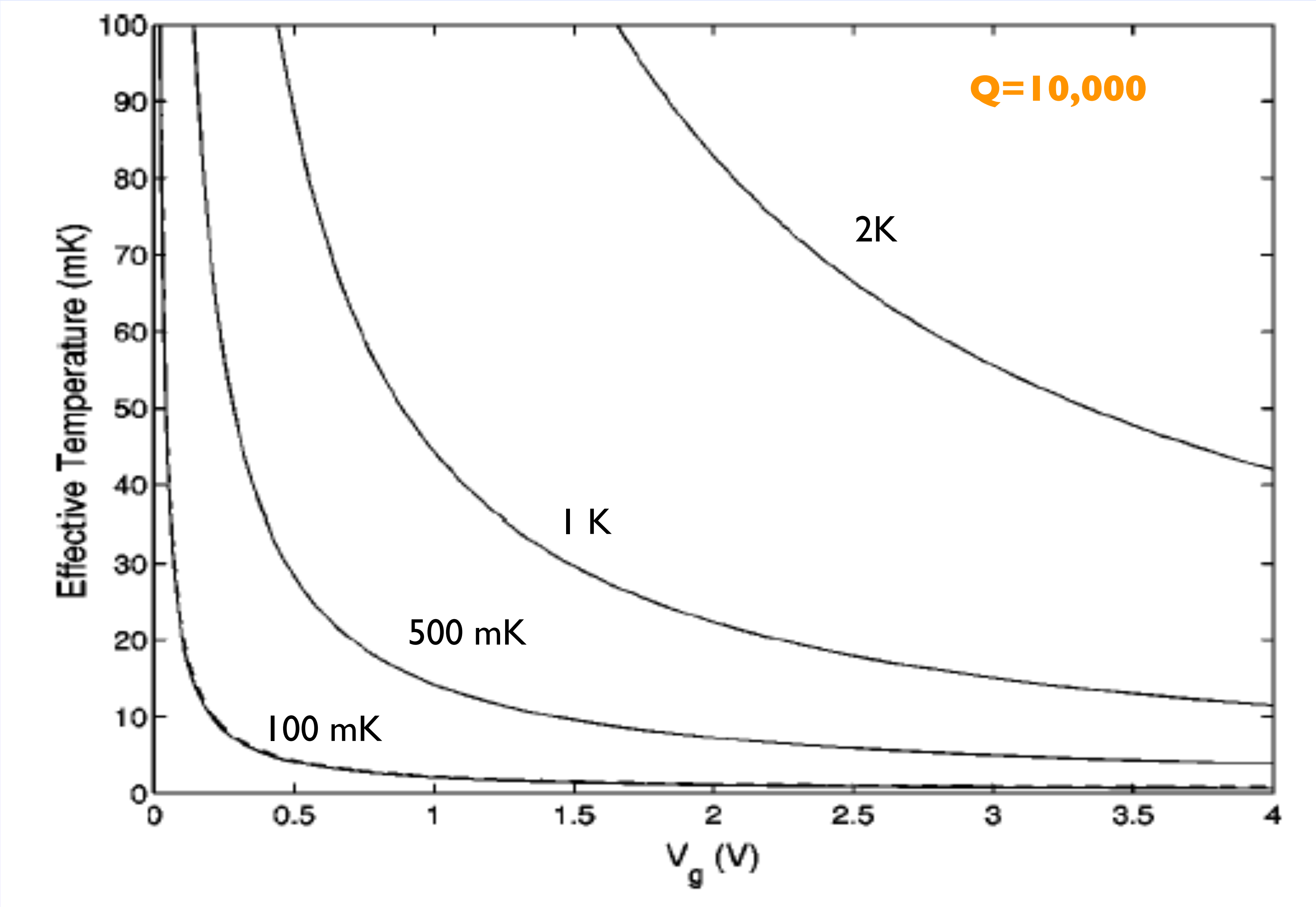
Can use standard LQG theory

Continuous X measurement of a Quantum harmonic oscillator	=	Continuous X measurement of a Classical harmonic oscillator + White noise

Because of this one can use classical LQG theory for linear systems
However, the noise depends on the accuracy of the measurement…
… and we must therefore optimize over the measurement as well.

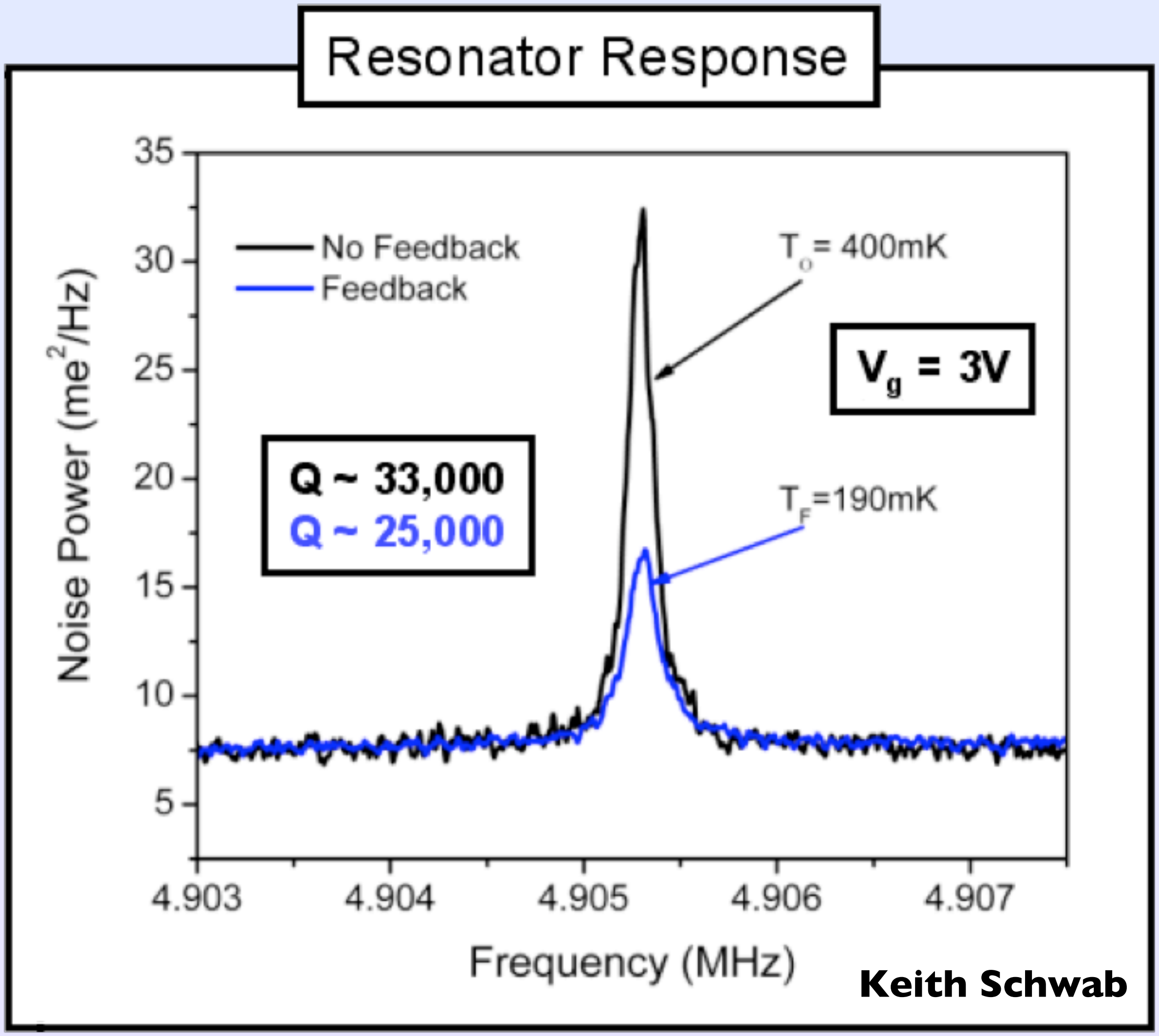
--- Belavkin ~ 1980s
--- Yanagisawa and Kimura (1998)
--- Doherty and Jacobs (1999)

Results



Q=10,000
2K
1 K
500 mK
100 mK
Effective Temperature (mK)
V_g (V)
100
90
80
70
60
50
40
30
20
10
0
0
0.5
1
1.5
2
2.5
3
3.5
4

Current Experiments in Schwab's Group



- Actively cooled the resonator from 400 mK to 190 mK.
- In this case the predicted final temperature was 220 mK.

Measurement and Non-linear Dynamics

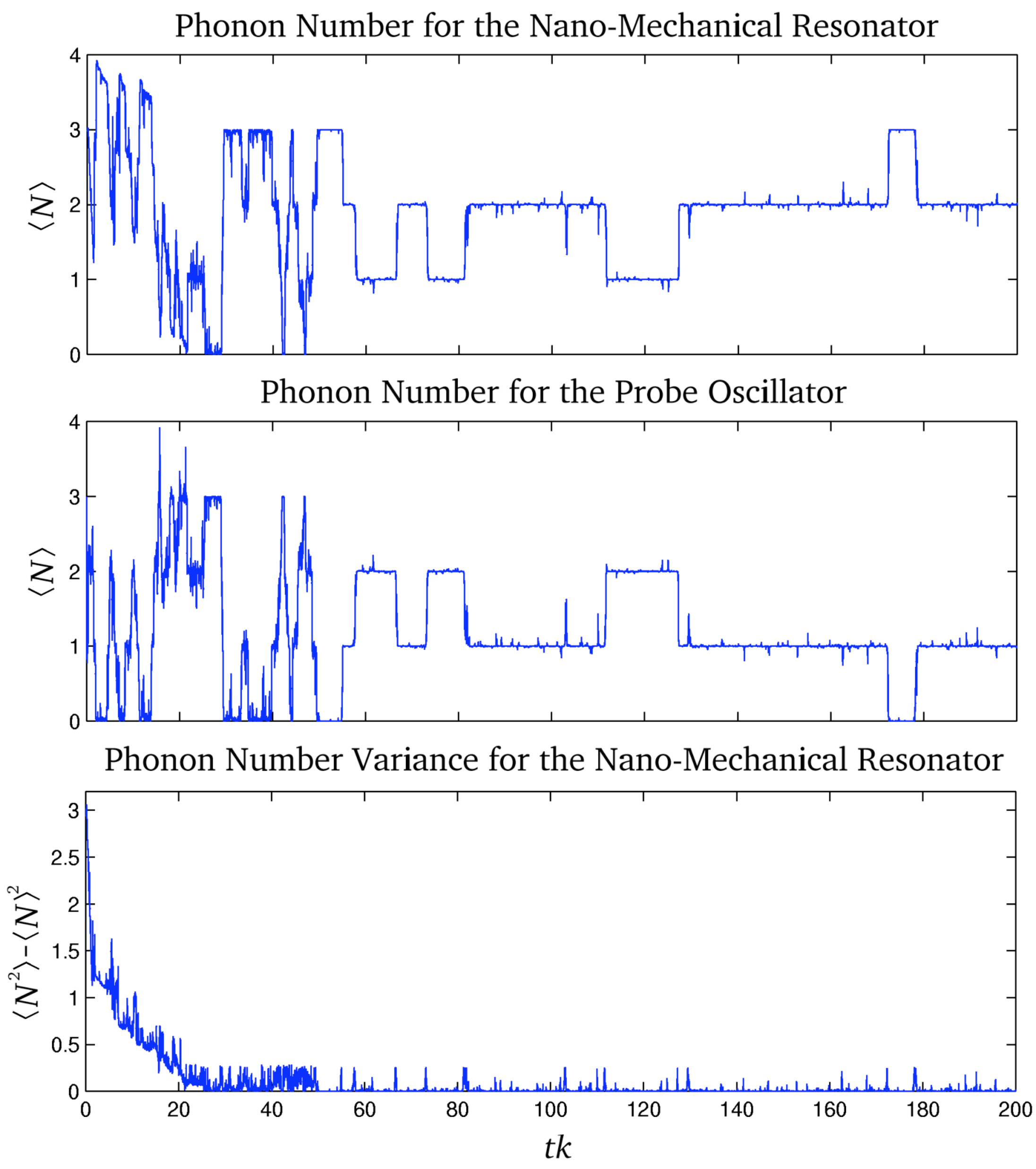
Consider now two coupled Harmonic oscillators.

Classically this is a simple linear system with a four dimensional phase-space.

If we monitor the position then this is also true for a quantum system.

However, if we monitor the energy the dynamics becomes non-linear!

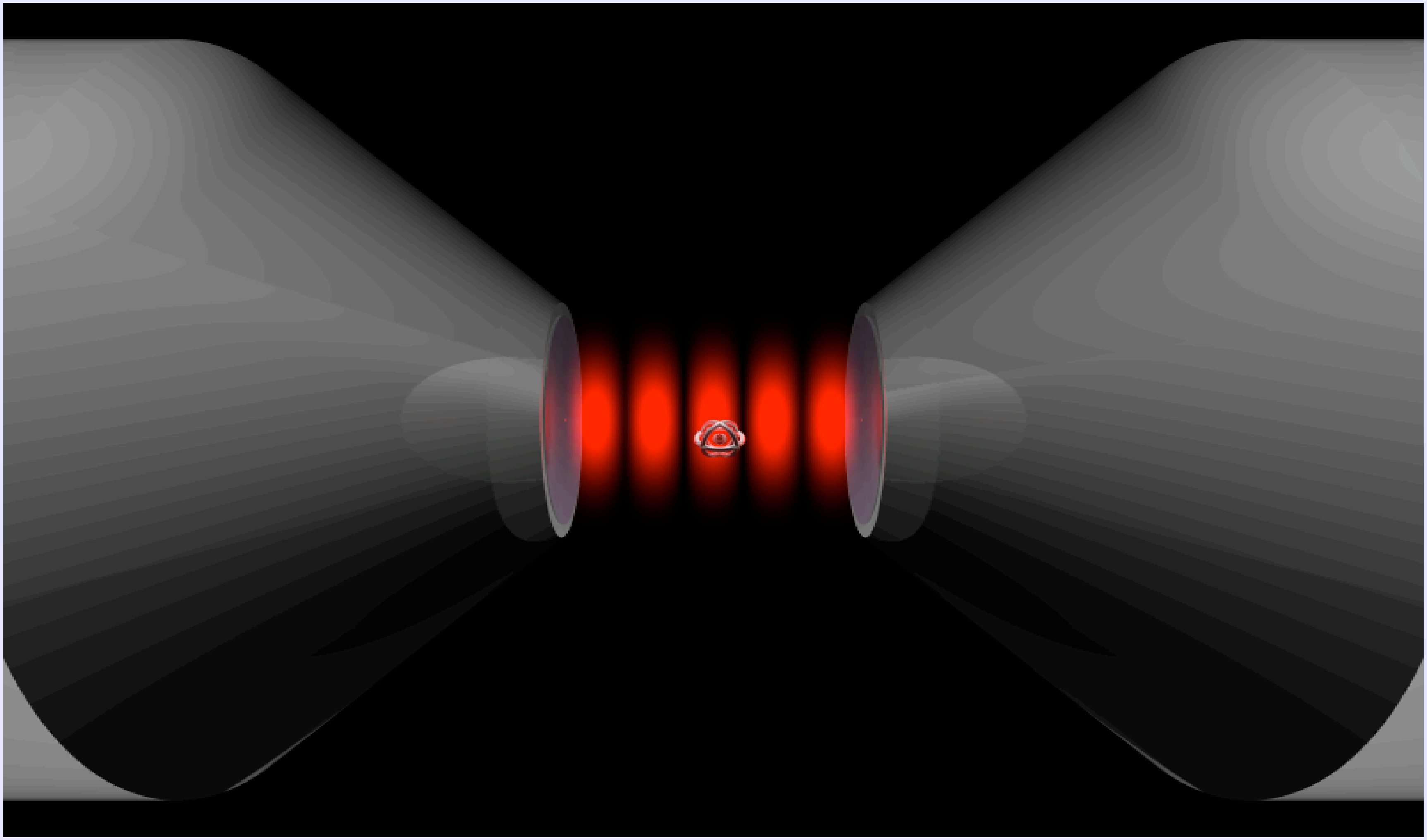
Since the energy states are discrete, the dynamics includes jumps in the energy



--- Lougovsky and Jacobs, quant-ph/0607102

Cooling an Atom in an Optical Cavity

--- Steck, Jacobs, Mabuchi, Bhattacharya and Habib, PRL 92, 223004 (2004)
& quant-ph/0509039



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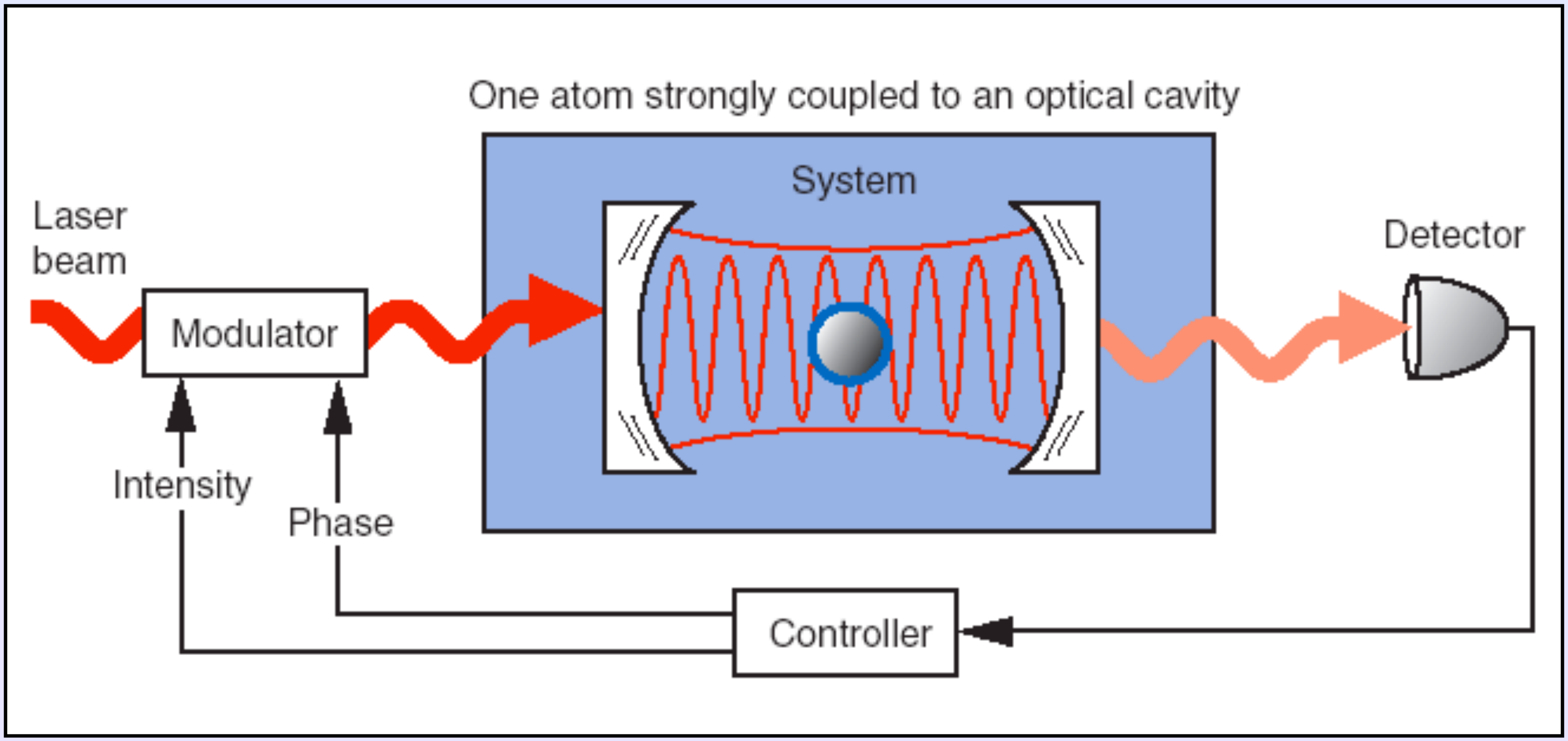
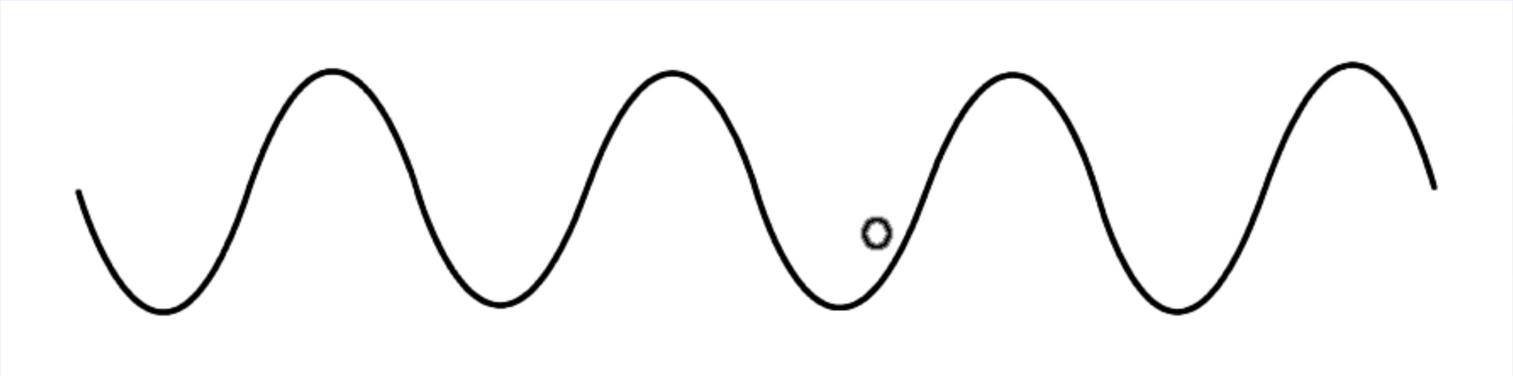


Tracking and Feedback Control

Feedback:

The atom sees a sinusoidal potential due to the standing wave

--- we can alter the potential by altering the intensity of the input laser

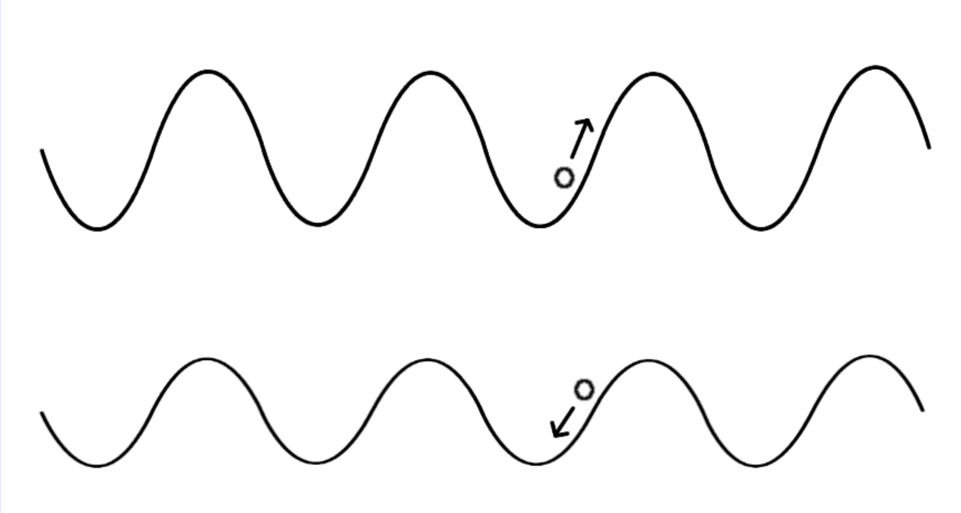


Tracking:

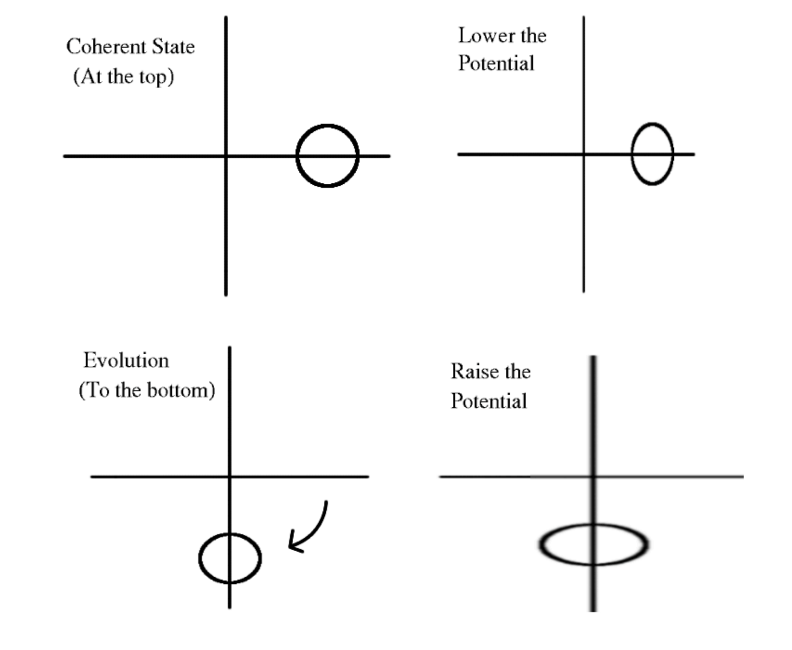
The atom affects the phase of the output light, and the amount of the phase shift depends on the position of the atom.

An Effective Feedback Algorithm

Simple algorithm:
Raise the potential when the atom is climbing;
Drop the potential when the atom is falling:



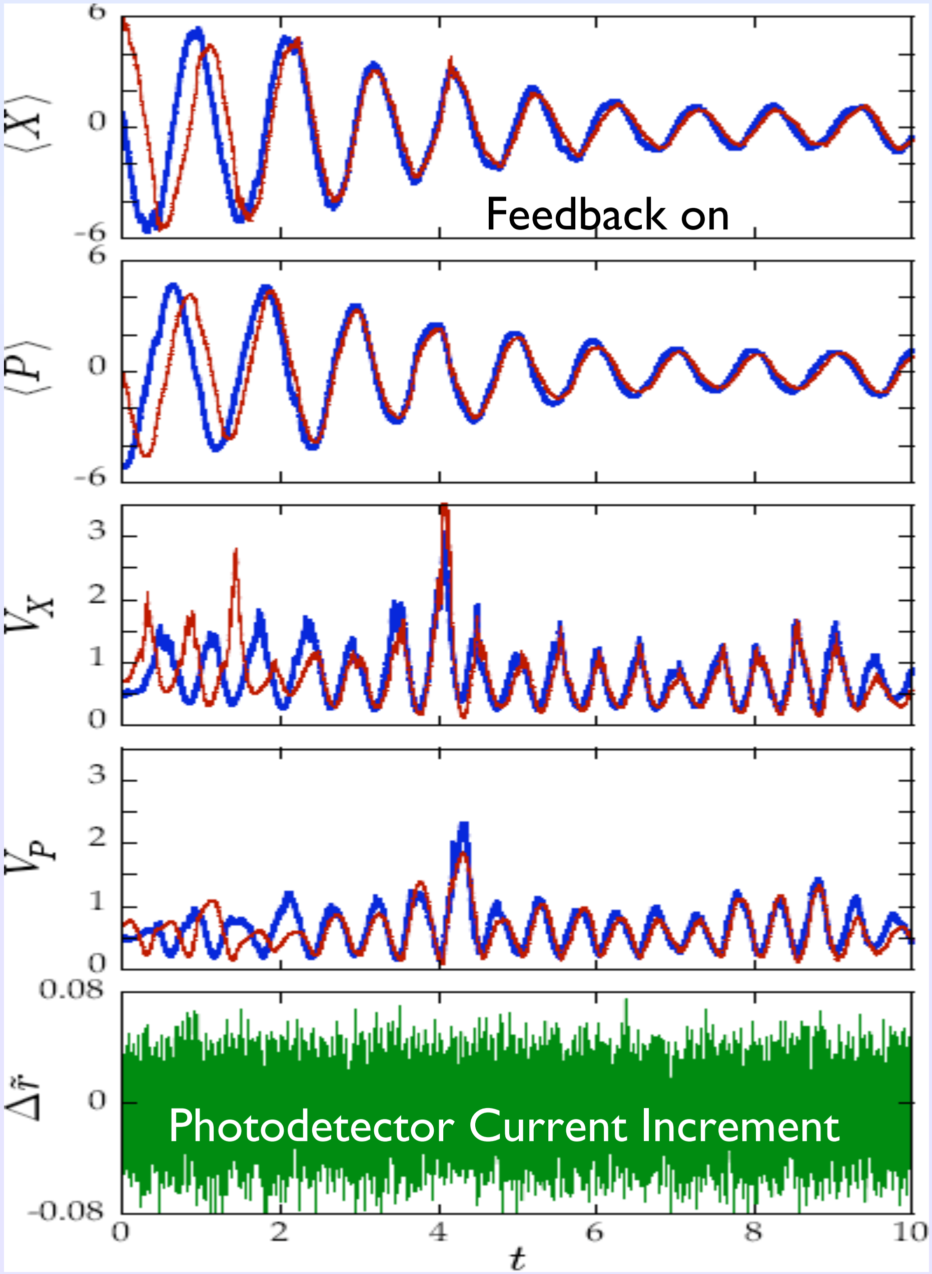
But this doesn't work:
It squeezes the state, feeding energy into the variances of the wavefunction

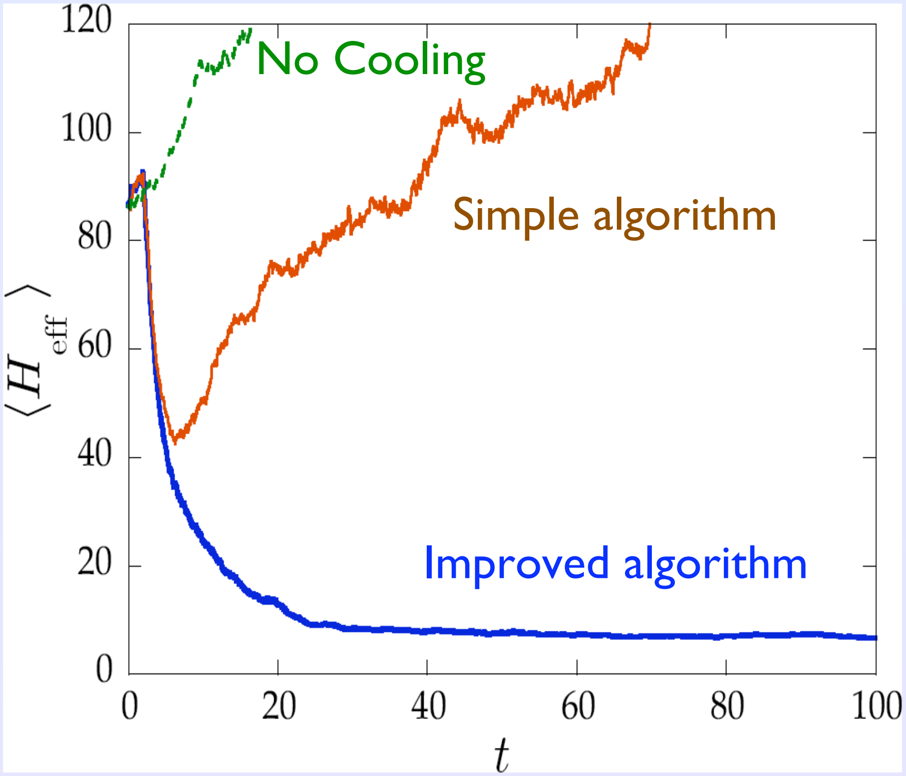


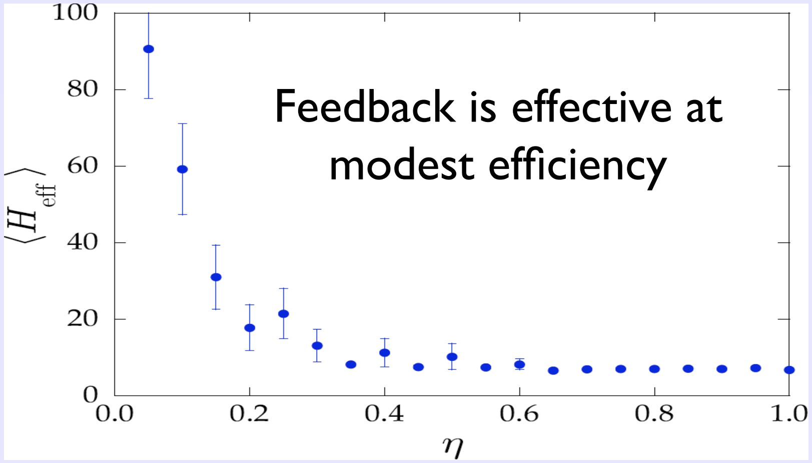
To obtain an effective algorithm we need to calculate the change of the total motional energy due to a change in the potential:

$$\partial_t \langle H_{\mathrm{eff}} \rangle_{\mathrm{fb}} = -(\partial_t V_{\max}) \langle \cos^2(kx) \rangle$$

Cooling Simulations

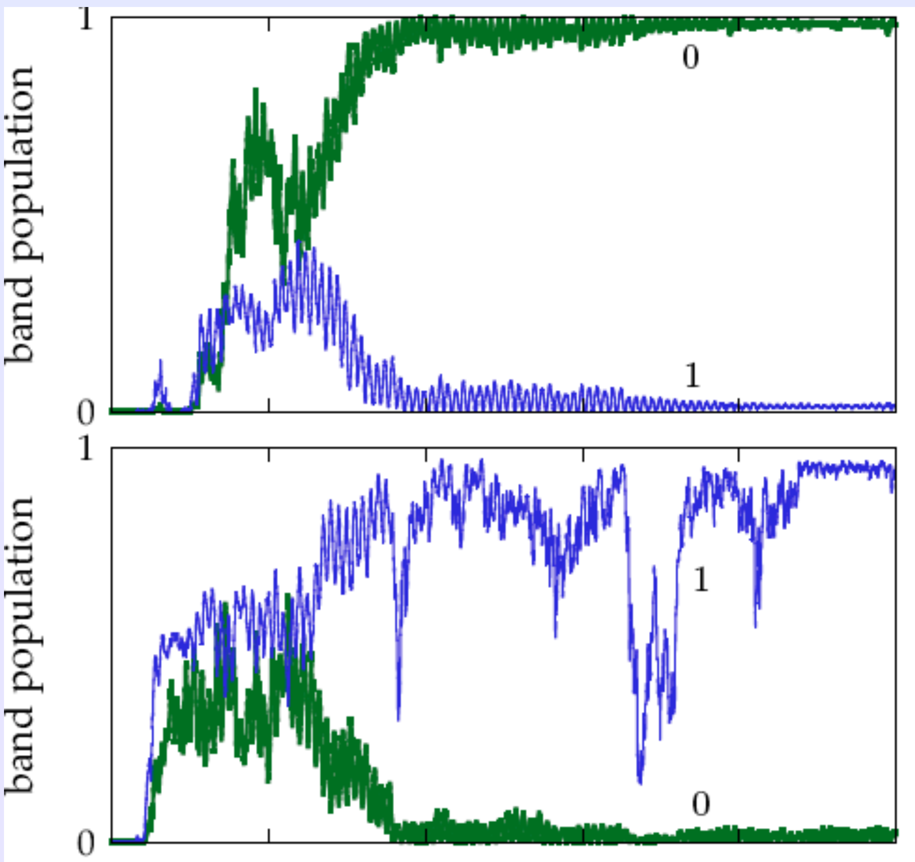


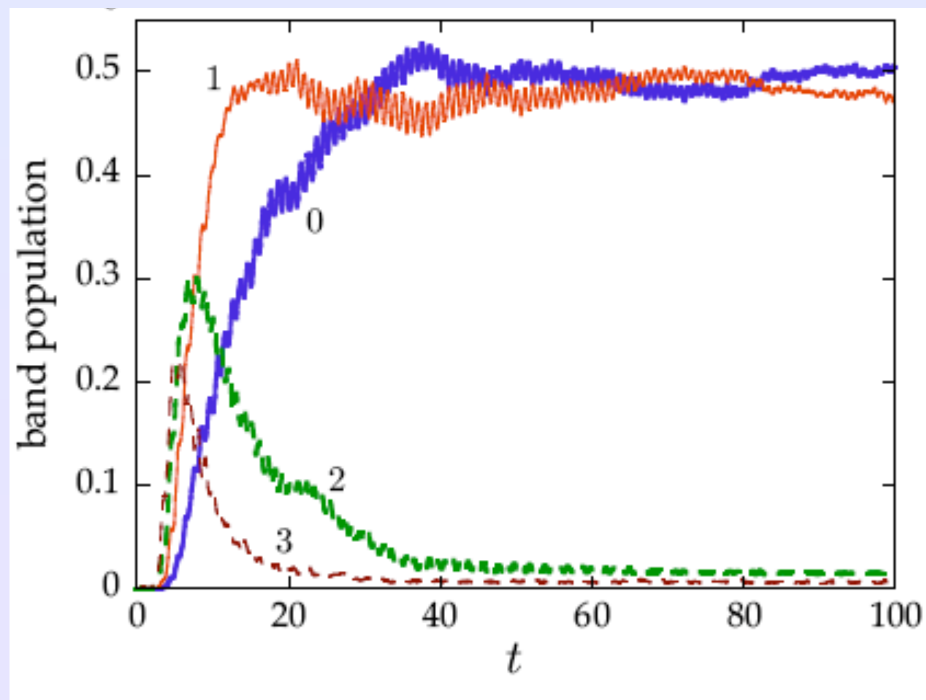




Cooling to a Single Pure State

The algorithm will cool to either the ground state or the first excited state.
But the measurement tells you which one you have got.





Adaptive Measurement

In adaptive measurement one changes the measurement via feedback as it proceeds.

Adaptive measurement is useful because in quantum systems:

1. Often quantum states cannot be fully distinguished
2. Measurements can disturb the quantity being measured
3. Availability of measurements is often restricted

Adaptive measurement widens the range of available measurements in a given situation, and is useful in precision measurement and communication.

Communication

The first use of quantum adaptive measurement was by Dolinar in 1973.

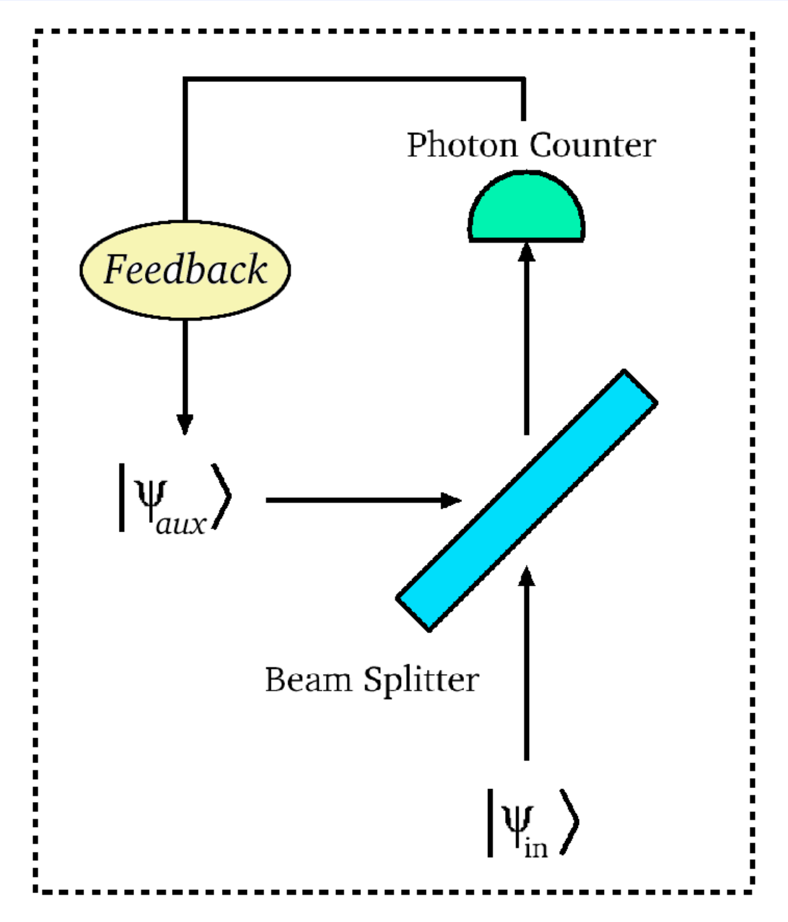
He considered a binary communication channel in which:

1 = weak coherent laser pulse
0 = no pulse

Measurement restriction: one can only count photons.

In Quantum Optics 1 and 0 cannot be unambiguously distinguished
--- there is always a finite chance the pulse will have 0 photons

The optimal measurement requires feedback after each photon detection!



Communication II

Dolinar’s scheme distinguishes maximally *well* between two states.

One might alternatively wish to ***distinguish maximally fast***.

It turns out that one can use feedback to increase the speed of discrimination when the two states are not completely distinguishable. (this is a quantum mechanical effect)

--- Jacobs, Quantum. Inf. Comp. (in press)

Another application of feedback is in ***measuring optical phase***.

Since we can only count photons, measuring phase is not trivial, and adaptive measurements are better than non-adaptive measurements.

--- Berry and Wiseman, PRA 65, 043803 (2002)
--- Pope, Wiseman and Langford, PRA 70, 043812 (2004)

Overview and Open Questions

Three broad areas

1. General results for optimal and robust control:
    - Obtaining exact results for classes of quantum systems.
    - Exploring the relationship between measurement and disturbance

2. Examining classes of applications unique to quantum systems
    - Various kinds of adaptive measurements

3. Designing control algorithms for applications in specific systems
    - Cooling nano-resonators, trapped atoms
    - Controlling the dynamics of coupled nanoscopic systems