

Known errata for *Stochastic Processes for Physicists*, as of Nov 25, 2015:

### Errata for all versions printed to-date:

Eq. (3.69): Should be

$$\frac{dV_x}{dt} = -2(\gamma - \beta^2)\langle x^2 \rangle + 2\gamma\langle x \rangle^2 = -2(\gamma - \beta^2)V_x + 2\beta^2\langle x \rangle^2.$$

Eq. (3.94) should be

$$\begin{aligned} I &= \frac{1}{2} \lim_{\Delta t \rightarrow 0} \sum_{n=0}^{N-1} (Z_n + f_n \Delta W_n)^2 - Z_n^2 - (f_n \Delta W_n)^2 \\ &= \frac{1}{2} \lim_{\Delta t \rightarrow 0} \sum_{n=0}^{N-1} \Delta(Z_n^2) - f_n^2 \Delta W_n^2 = \frac{1}{2} \left( \int_0^T d(Z_n^2(t)) - \int_0^T f^2(t) (dW)^2 \right) \\ &= \frac{1}{2} \left( Z^2(T) - \int_0^T f^2(t) dt \right). \end{aligned} \tag{1}$$

Eq.(2.9) should be

$$d \begin{pmatrix} x \\ p \end{pmatrix} \equiv \begin{pmatrix} dx \\ dp \end{pmatrix} = \begin{pmatrix} p/m \\ -kx \end{pmatrix} dt = \begin{pmatrix} 0 & 1/m \\ -k & 0 \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix} dt.$$

Eq.(5.39) should be

$$\bar{a}(\Delta t) = \frac{1}{m} \left( \frac{\Delta p}{\Delta t} \right) = \frac{1}{m} \left( -\gamma p + \beta \frac{\Delta W}{\Delta t} \right)$$

Eq.(7.28): remove the factor of “2” in front of the  $J$ .

Eq.(7.29): remove the factor of “2” on the bottom line.

Eq.(7.30): replace the minus sign with a plus sign.

Eq.(7.33): In the second line of this equation, the expression  $\gamma \partial P / \partial p$  should be  $\gamma \partial / \partial p$ .

Section 7.8: The definition given for the diffusion coefficient  $D$  is non-standard. This definition makes  $D$  equal to twice the value of the standard definition. That is, the standard definition is to write the diffusion equation without the factor of  $1/2$ .

### Errata for the first and second printings:

(these have been corrected in the newer print-on-demand version)

The heading of section 1.5 should be “Adding independent random variables together”.

The first line of section 1.5: “When we have two continuous random variables ...” should be “When we have two independent continuous random variables ...”

Eq.(1.15) should be

$$P_Z(z) = \int_{-\infty}^{\infty} P_X(z-s)P_Y(s)ds \equiv P_X * P_Y$$

The first sentence of section 1.6: “... it can be useful to know how calculate...” should be “... it can be useful to know how to calculate...”

In section 1.9: the text “is defined as the nth derivative of the log of the characteristic function, also evaluated at zero.” should be replaced with “is also given by Eq.(1.42), but with  $\chi$  replaced with  $\log(\chi)$ ”.

## Ch 2

Eq.(2.4) should be

$$\frac{dx}{dt} = \frac{p}{m} \quad \text{and} \quad \frac{dp}{dt} = -kx.$$

Eq.(2.5) should be

$$\frac{d}{dt} \begin{pmatrix} x \\ p \end{pmatrix} \equiv \begin{pmatrix} dx/dt \\ dp/dt \end{pmatrix} = \begin{pmatrix} p/m \\ -kx \end{pmatrix} = \begin{pmatrix} 0 & 1/m \\ -k & 0 \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}.$$

Eq.(2.5) should be

$$A = \begin{pmatrix} 0 & 1/m \\ -k & 0 \end{pmatrix},$$

The minus sign in front of the  $\gamma$  in Eq.(2.50) should be plus.

Eq.(2.53) should be

$$e^{\lambda\sigma} = \cosh(\lambda)I + \sinh(\lambda)\sigma,$$

## Ch 3

Eq.(3.28) should be

$$P(Z_n) = \frac{e^{-Z_n/(2\Delta t)}}{\sqrt{2\pi\Delta t Z_n}}.$$

Eq.(3.32) should be

$$\frac{1}{\sqrt{1-x}} \approx 1 + x/2,$$

Eqs. (3.39) and (3.40): all occurrences of  $f$  should be replaced by  $y$ .

Eq. (3.46): last line should be

$$= g^2 \int_0^t e^{2\gamma s} ds = \frac{g^2}{2\gamma} (e^{2\gamma t} - 1).$$

Second line after Eq.(3.50): “Section 1.2” should be “Section 2.4”

Eq. (3.62): The “ $dt$ ” on the right hand side should be deleted.

Eq. (3.67): Should be

$$d\langle x^2 \rangle = -(2\gamma - \beta^2)\langle x^2 \rangle dt.$$

Eq. (3.68): Should be

$$\frac{dV_x}{dt} = \frac{d\langle x^2 \rangle}{dt} - \frac{d\langle x \rangle^2}{dt} = \frac{d\langle x^2 \rangle}{dt} - 2\langle x \rangle \frac{d\langle x \rangle}{dt}.$$

Last paragraph of section 3.7: “... will include the third moments, and so.” should be “... will include the third moments, and so on.”

Eq. (3.74): the second line should be

$$= \left\langle M \begin{pmatrix} dW_1 \\ dW_2 \end{pmatrix} (dW_1, dW_2) M^T \right\rangle$$

Eq. (3.76): the second line should be

$$= M \begin{pmatrix} dW_1 \\ dW_2 \end{pmatrix} (dW_1, dW_2) M^T$$

Eq. (3.75): should be  $C = 2\eta\sqrt{1-\eta^2}$

Eq. (3.89) Should be

$$I = \int_0^T \int_t^T ds f(t) dW(t) = \int_0^T (T-t)f(t) dW(t)$$

Eq. (3.90) should be

$$I = \int_0^T [(T-t)f(t)]^2 dt$$

Section 3.9, second paragraph: “equation drive by” should be “equation driven by”

Page 53, exercise 9, two lines below Eq. (3.127): “fist” should be “first”

#### Ch 4

Page 55, 4th line from the bottom: “pot” should be “plot”

Eq. (4.17) should be

$$X(t) = x' + \int_{t'}^t dW = x' + W(t) - W(t'),$$

Eq. (4.21) should be

$$P(x, t; x', t') = P(x, t|x', t')P(x', t') = \frac{e^{-(x-x')^2/[2(t-t')] - x'^2/[2t']}}{2\pi\sqrt{(t-t')t'}},$$

First line of section 4.5: the two occurrences of “Section 1” should be replaced with “Section 1.8”

Eq.(4.35), last line:  $h(t, \tau)$  should be replaced with  $h(\tau)$

Eq.(4.36):  $h(t, \tau)$  should be replaced with  $h(\tau)$

Eq.(4.39):  $dt$  should be replaced with  $d\nu$

Eq.(4.54): The lower limit of the integral is currently  $\infty$ . It should be replaced by  $-\infty$

#### Ch 5

Eq. (5.12) should be

$$\begin{aligned} x(t) &= \int_0^t \frac{p(s)}{m} ds = \frac{p(0)}{m} \int_0^t e^{-\gamma s} ds + \frac{g}{m} \int_0^t \left[ \int_0^s e^{-\gamma(s-s')} dW(s') \right] ds \\ &= \frac{p(0)}{m} \int_0^t e^{-\gamma s} ds + \frac{g}{m} \int_0^t e^{\gamma s'} \left[ \int_{s'}^t e^{-\gamma s} ds \right] dW(s') \\ &= \frac{1}{m\gamma} (1 - e^{-\gamma t}) p(0) + \frac{g}{m\gamma} \int_0^t (1 - e^{-\gamma(t-s')}) dW(s') \\ &= \frac{1}{m\gamma} (1 - e^{-\gamma t}) p(0) + \frac{g}{m\gamma} \int_0^t (1 - e^{-\gamma s}) dW(s) \end{aligned}$$

#### Ch 6

All occurrences of “Milstien’s method” (pages 95 and 98) should be replaced by “Milstein’s method”

In the heading of section 6.5, “Runge-Kutter” should be “Runge-Kutta”.

## Ch 7

Eq.(7.2): The RHS should be  $dh$ , not  $df$ .

Just after Eq.(7.2): “. . . equation for the mean of  $f$  . . .” should be “. . . equation for the mean of  $h$  . . .”

Just before Eq.(7.22): “differential equation for  $h$ ” should be “differential equation for  $\xi$ ”

Eq.(7.23) should have a “2” inserted before the  $f(u)$ .

Eq.(7.28) should have a “2” inserted before the  $f(u)$ .

## Ch 8

In the paragraph before Eq.(8.4), the expression  $(\lambda\Delta t)^m(1-\Delta t)^{M-m}$  should be  $(\lambda\Delta t)^m(1-\lambda\Delta t)^{M-m}$ .

In Eqs. (8.35) and (8.36), all occurrences of  $\lambda^n$  should be replaced with  $(\lambda t)^n$ .

## Ch 9

In Eq.(9.16),  $f(t)$  should be  $f(s)$ .

In Eq.(9.17),  $[f(t)]^\alpha dt$  should be  $[f(s)]^\alpha ds$ .

In Eq.(9.44),  $g(x, t)$  should be  $g^2(x, t)$ .

Eq.(9.46) should be

$$dy(t) = \left(\frac{\partial y}{\partial x}\right) dx_c(t) + \left[\left(\frac{\partial y}{\partial t}\right) + \frac{g(x, t)}{2} \left(\frac{\partial^2 y}{\partial x^2}\right)\right] dt + [y(x + \Delta J(t), t) - y(x, t)],$$

$$\text{with } \Delta J(t) = \begin{cases} \text{size of jump at } t_i, & t = t_i, \forall i \\ 0 & \text{otherwise} \end{cases}$$

Eq.(9.47) should be

$$dy(t) = \left(\frac{\partial y}{\partial x}\right) dx(t) + \left[\left(\frac{\partial y}{\partial t}\right) + \frac{g(x, t)}{2} \left(\frac{\partial^2 y}{\partial x^2}\right)\right] dt + \left[y(x + \Delta J(t), t) - y(x, t) - \frac{dy(x, t)}{dx} \Delta J(t)\right]$$

First line after Eq.(9.47) should be: “In this expression the term  $-\frac{dy}{dx} \Delta J(t), \dots$ ”

**Ch 10**

Three lines above Eq. (10.20): "... above exert in plan language" should be "... above excerpt in plain language"

**Errata for the first printing only:****Ch 1**

Page 2: Figure 1 illustrates summing the probabilities from 3 to 4, not 4 to 6 as implied in the text.

**Appendix**

Page 181, Eq.(A5): The last term on the second line should not have a minus sign.

Pages 181 and 182, Eqs. (A6) - (A11): Wherever the expression  $-\beta^2/(4\alpha^2)$  appears in an exponential, it should be replaced with  $+\beta^2/(4\alpha)$ .

Eqs.(A.18) and (A19) should be

$$\int_0^\infty x^{2n} e^{-\alpha x^2} dx = \frac{\sqrt{\pi}}{(2\sqrt{\alpha})^{2n+1}} \frac{(2n)!}{n!},$$
$$\int_0^\infty x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}}.$$

**Ch 3**

Eq. (3.78): ( $M\mathbf{dV}$ ) should be replaced with ( $M\mathbf{dW}$ ).