What’s Quantum about Quantum Feedback Control?

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Controlling Systems with Feedback

Sound and Parameter Uncertainties

Inputs → Dynamical System → Outputs

Controller

Measurement Errors
Feedback in Quantum Systems...

Measurements **change** the dynamics of a system, and introduce **noise**.

This means that...

**One.** Must include measurements when optimizing control

**Two.** Must choose our measurements carefully: New applications for feedback: **adaptive measurements** to reduce noise and optimize information extraction.

**Three.** The complex interaction between quantum measurements and dynamics also leads to other effects...
Quantum Feedback: Applications

1. Quantum versions of traditional applications...

   Stabilization/Noise Reduction
   Cooling a system to its ground state
   Quantum Error-Correction

2. Adaptive measurements for...

   Communication
   Precision measurement
   Speeding up State-Collapse/Preparation

3. And even using measurements for control...
Cooling a Nano-Mechanical Resonator

A nano-resonator

Specs:
- Frequency ~ 100 MHz
- Quality factor ~ $10^5$


* asterisks denote authors who are here at the workshop
It turns out that...

A Quantum Oscillator + Position measurement + Linear feedback forces
equivalent to
A Classical Oscillator + Position Measurement + Linear feedback forces + Extra white noise

So we can use standard optimal control theory for linear systems to cool and control quantum oscillators.

— Yanagisawa & Kimura, in Learning, Control and Hybrid Systems (Springer, 1998)
Optimal Linear Control for other systems?

No!

All other quantum systems, when measured, are nonlinear. Very few optimal results exist for nonlinear systems.

There is one known partial exception:

For a single qubit, in the “regime of good control” (when the control keeps it near the target-state) the motion is nearly linear

Quantum States of Nano-Resonators

When a resonator is really cold we can create interesting states...

We can create a “quantum superposition” in which the resonator is in “two places at once”

Can do this by continuously measuring the square of a resonator’s position.

Quantum States of Nano-Resonators

The state of a quantum system can be represented by a “quasi”-probability density over phase-space, \( W(x,p) \). This is a quasi-probability because it can be negative. The marginals of \( W(x,p) \) are the real probability densities for \( x \) and \( p \). \( W(x,p) \) is called the Wigner Function.

A localized ‘classical’ state

A superposition state
Creating a Superposition (Cat) State

Movie of a continuous measurement creating a ‘cat’ state:

Creating a Superposition (Cat) State

Some Snapshots of creating a ‘cat’ state:

Spins/Qubits – The Bloch Sphere

A single spin-$1/2$ can point in any direction, but has only two states in a given direction - when measured along one direction, it will either be found to point in that direction, or directly opposite.

The “Bloch-sphere” is a way of representing the spin state. If the state of the spin is completely known, then the “Bloch-vector” has length 1 and gives the direction of the spin.

If the spin state is not completely known, then the Bloch-vector is shorter. If the spin direction is completely unknown, then the Bloch vector has no length, and so no direction.
Quantum measurements change the state of a system. Because of this, quantum measurements provide information about two separate things — the initial state, and the final state. Below is an example in which an initial state of a qubit is strongly measured in the $x$-direction. The final state is completely determined by the measurement, but only partial information is provided about the initial state.
Continuous Measurements on Qubits

In feedback control one usually considers continuous measurements. For short time-intervals these only extract a little bit of information about the state of the system. If we start in a $+z$ state, then the effect of a continuous $x$-measurement for a short period of time is:

**Equally-likely outcomes**

**Final State**

**Measurement Direction**

**Equally-likely outcomes**

**Final State**
Two Purely Quantum Effects

The dynamics introduced by quantum measurements (the quantum back-action) leads to some remarkable effects. We will describe two here:

1. Feedback can be used to increase the speed at which the measurement “collapses” the system to a final state.


2. The random quantum back-action of adaptive measurements can be used to control a quantum system

   (This is hot off the press - was posted on monday!)
The closer the Bloch-vector to the surface of the sphere, the more certain we are of the spin direction:

1. It turns out that measuring in a direction orthogonal to the spin direction lengthens the Bloch vector faster. This is a purely quantum effect.

2. But this changes the direction of the Bloch vector.

3. So we have to continually change the direction of the measurement to make the state “collapse” fastest to a single certain direction. The diagram illustrates this:
This is remarkably simple. The random back-action of the measurement causes diffusion on the surface of the Bloch sphere. By using feedback to control the rate and the direction of the measurement, we can set up a diffusion gradient. Diffusion gradients act similarly to forces - a system will move in the direction in which the diffusion decreases. If the measurement is strong, this can be used to obtain very accurate control.
Some introductory articles are:

Jacobs, Proceedings of ASCC06 p.35 (2006)

Habib, Jacobs, and Mabuchi, Los Alamos Science 27, 126 (2002)
downloadable at: http://www.quantum.umb.edu/Jacobs/QFC.html

Jacobs & Shabani, Contemporary Physics 27, 126 (2008)