Measurement Back-Action
Exploiting It & Avoiding It

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Overview

Part 1 **Feedback Control via Diffusion Gradients**

i) Dynamics of diffusion gradients

ii) Controlling a single q-bit

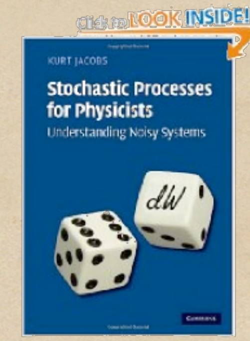
Part 2 **Feedback Control of Schrödinger Cats**

Justin Finn & Kurt Jacobs

i) x^2 measurements and cat states

ii) Gaussian estimators and control tools

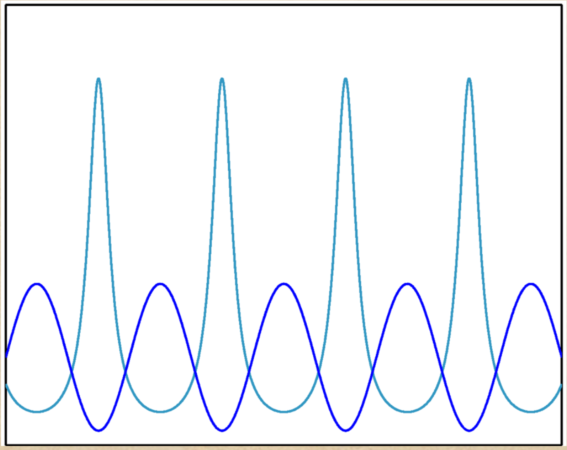
Better Together?...



Diffusion Gradients

The steady-state probability density due to spatially varying diffusion

$$P(x) = \frac{1}{\mathcal{N} D(x)}$$



Diffusion Gradients

We can see that diffusion gradients act like deterministic motion by calculating the probability current...

Diffusion and deterministic motion:

$$dx = v dt + \sqrt{D(x)} dW.$$

The equivalent Fokker-Planck Eq:

$$\frac{\partial P}{\partial t} = -v \frac{\partial P}{\partial x} + \frac{1}{2} \frac{\partial^2}{\partial x^2} [D(x) P]$$

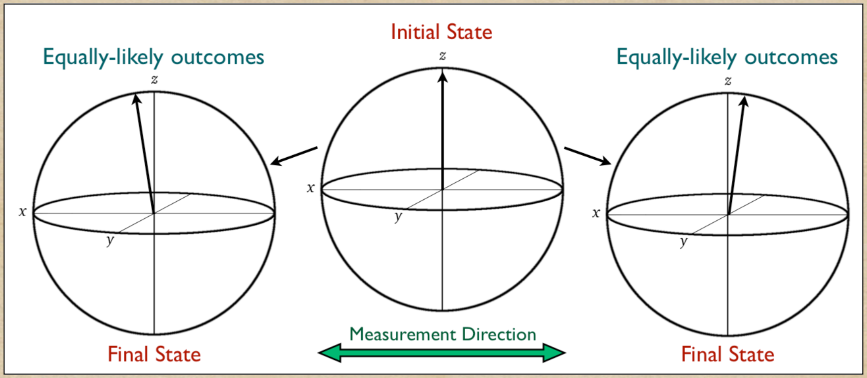
The probability current J is given by:

$$-\frac{\partial J}{\partial x} = \frac{\partial P}{\partial t}$$

And the expression for J is:

$$J(x) = \left(v - \frac{1}{2} \frac{\partial D}{\partial x} \right) P - \frac{D(x)}{2} \frac{\partial P}{\partial x}.$$

Measurements and Diffusion

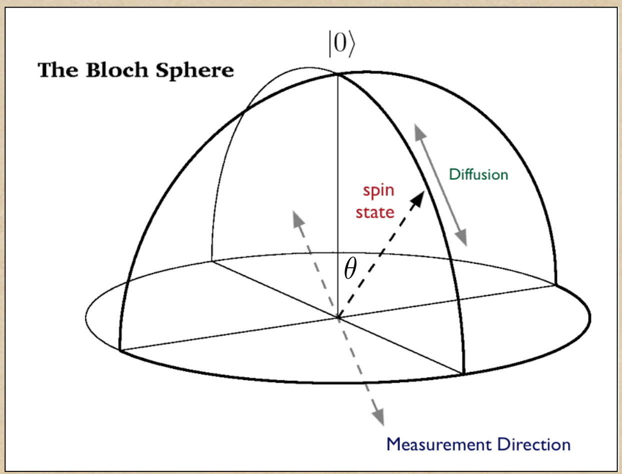


Continuously measuring in a basis perpendicular to the Bloch vector generates diffusion on the Bloch sphere.

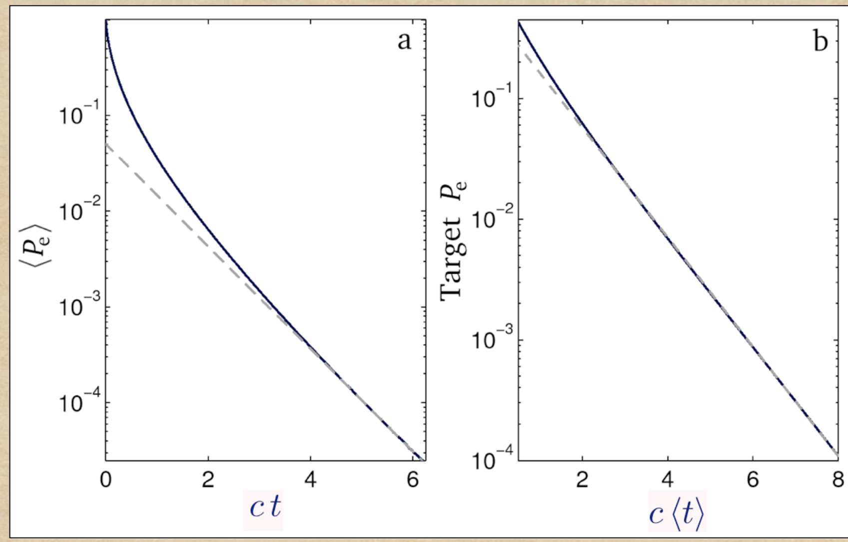
Control Algorithm

We make the measurement strength k depend on θ:

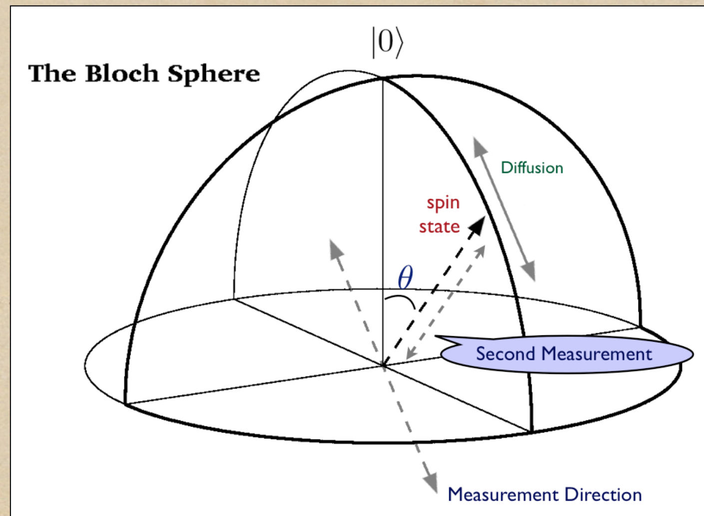
$$k = \kappa\theta^2$$



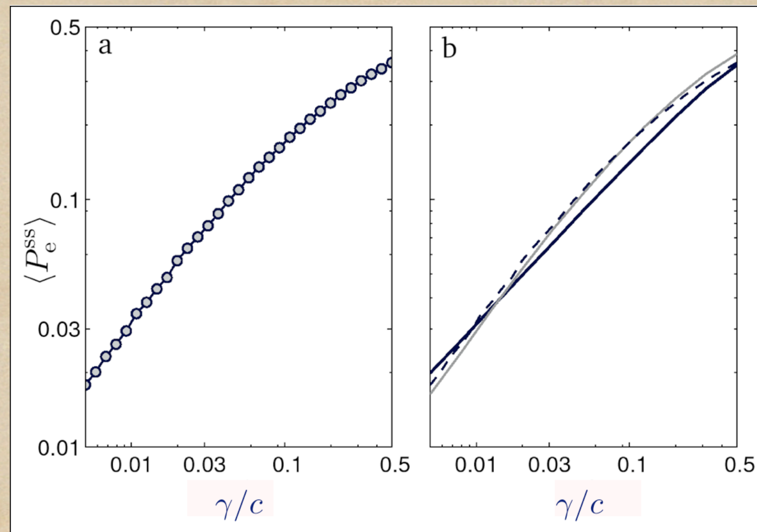
Performance



Feedback Control with Noise



Steady-State Performance

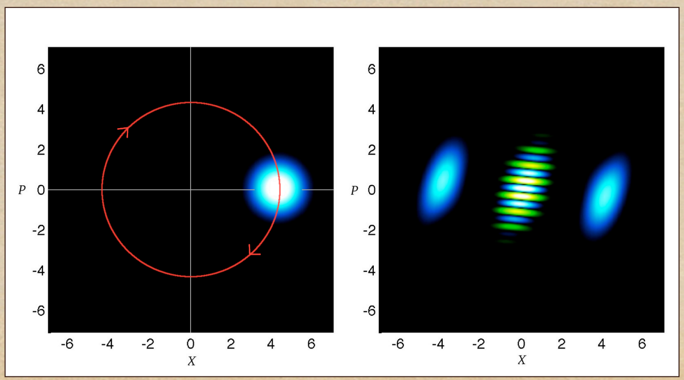


Feedback Control of Schrödinger-Cats

Justin Finn & Kurt Jacobs

To Control Cats via Feedback we need:

i. A measurement that wont destroy them
ii. Feasible ways to track them in real-time
iii. Ways to control both components



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Ingredient I: Quadratic Measurements

A measurement of $\boldsymbol{x}^2$ generates cat-states...

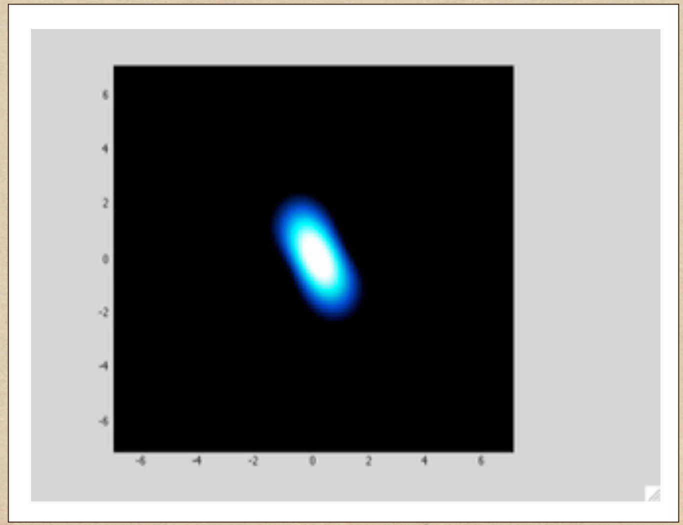
*– Jacobs, Tian, Finn, PRL **102**, 057208 (2009)*

(this means that cats preserve their symmetry in $\boldsymbol{x}$ as they evolve)

So if a cat is symmetric about $\boldsymbol{x} = \boldsymbol{x}_0$
we can preserve the cat by measuring $(\boldsymbol{x} - \boldsymbol{x}_0)^2$

Monitoring X^2

So here is what happens...



Quadratic Measurements of X

Require a coupling proportional to $(x - x_0)^2$
Can obtain this by extending the perturbative technique in

*– Jacobs & Landahl, PRL **103**, 067201 (2009)*

We couple a single qubit perturbatively to the resonator via

$$\lambda \sigma_z x$$

and tune the free Hamiltonian of the qubit to give

$$H = \Delta \sigma_x + \lambda \sigma_z (x - a)$$

Time-independent perturbation theory then gives us

$$H_{\text{eff}} = \Delta \sigma'_x \left[\frac{1}{2} + \varepsilon^2 (x - a)^2 + \ldots \right]$$

where $\varepsilon = \lambda / \Delta$

Ingredient II: A Gaussian Estimator

The measurement cant tell the difference between a **single blob** and **two blobs** symmetrically placed about ***x = a***

$$d\langle x\rangle = \frac{\langle p\rangle}{m}dt + 4\sqrt{2k}V_x\left(\langle x\rangle - a\right)dW$$

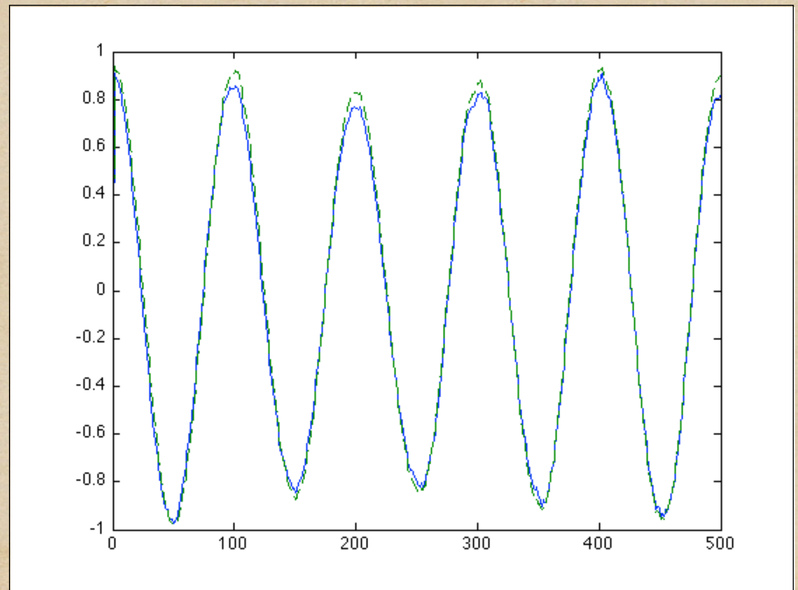
$$d\langle p\rangle = -m\omega^2\langle x\rangle dt + 4\sqrt{2kC_{xp}}\left(\langle x\rangle - a\right)dW$$

$$dV_x = \frac{2}{m}C_{xp}dt - 32kV_x^2\left(\langle x\rangle^2 + a^2\right)dt - 2\sqrt{2k}V_x^2 dW$$

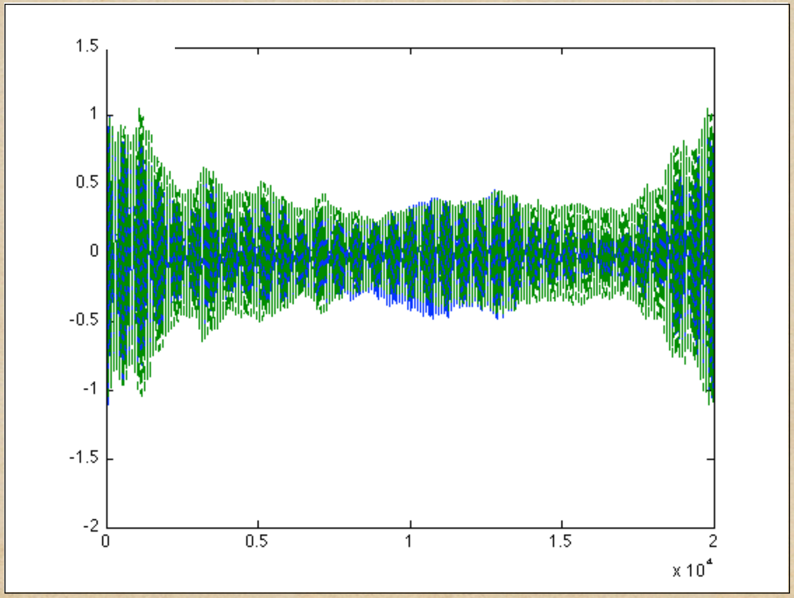
$$dV_p = -2m\omega^2 C_{xp}dt + 8k\left(\hbar^2 - 4C_{xp}^2\right)\left(\langle x\rangle^2 + a^2\right)dt - 2\sqrt{2k}V_xV_p dW$$

$$\begin{aligned} dC_{xp} &= \left(\frac{V_p}{m} - V_x m\omega^2\right)dt - 32kV_xC_{xp}\left(\langle x\rangle^2 + a^2\right)dt \\ &\quad + 2\sqrt{2k}\left(2\langle x\rangle\langle p\rangle V_x - \langle x\rangle^2 C_{xp} - V_xC_{xp}\right)dW \end{aligned}$$

Gaussian Estimator



Gaussian Estimator

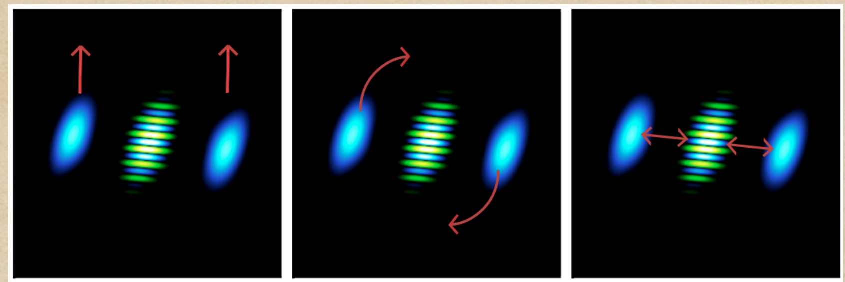


Ingredient III: Controls

Ideally we would like simple, linear controls to manipulate both components of the Schrödinger cat.

We can do *almost* that well...

Apply a linear Force **Change the Frequency** **?**



The measurement heats the system, increasing the inter-blob distance.
We can use the **cooling algorithm** in ***Steck et al., PRL 92, 223004 (2004)*** to reduce the distance between the blobs.

Summary

Feedback Control via Diffusion Gradients

Diffusion gradients induce effective drift in closed spaces
This can be exploited to control quantum systems
This is surprisingly effective

Feedback Control of Schrödinger Cats

x^2 measurements can be used to monitor cats Gaussian
estimators provide rapid tracking
linear Hamiltonians are sufficient for control